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ABSTRACT

The contents of this guide have been organized under five major topics: (1) number and operations; (2) sets, functions, relations, systems, and logic; (3) geometry; (4) measurement and estimation; and (5) selected topics. A scope-and-sequence chart is given for each of the topics for grades K-8. Behavioral objectives, teaching aids, and suggestions are listed for each of the topics at every grade level from K-8. A list of 33 references on problem solving is included. (MP)

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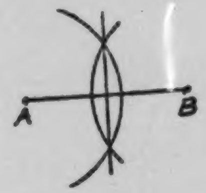

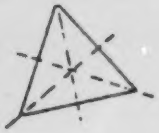

MATHEMATICS K-8

DEPARTMENT OF EDUCATION
STATE OF IDAHO

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IDAHO CURRICULUM GUIDE IN MATHEMATICS

GRADES K-8

IDAHO STATE DEPARTMENT OF EDUCATION

DIVISION OF INSTRUCTION

September 1970

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FOREWORD

Guides and materials in various subject areas are developed by the State Department of Education in an effort to promote and effect better methods that will aid students and teachers. When change necessitates curriculum modification, we utilize tested, proven and relevant methods and materials to accomplish current objectives.

The Idaho State Board of Education, in a formal action January 10, 1974, passed a resolution requiring instruction in the metric system in the public schools of Idaho beginning in 1974-75 school year, and conversion to the metric system by 1984. Nationwide observance of the metric system follows President Ford's signing of the Metric Conversion Act in December, 1975.

The Idaho Curriculum Guide in Mathematics has been revised to include the changes from the conventional system to the metric system. It is hoped that these revised guidelines will fit the changing needs of today's students.

Jerry L. Evans

JERRY L. EVANS

State Superintendent of Public Instruction

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COMMITTEE

Grateful acknowledgment is made to the following members of the Mathematics Curriculum Guide Committee. Many hours were devoted to the task of preparing material for the guide. A special thanks is extended to the committee members' administrators for releasing them from their duties to work on the mathematics guide.

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MATERIALS

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Major Portions of the *Clark County School District Curriculum Guideline, K-6* were used in the body of the Idaho Curriculum Guide in Mathematics.

BLOOMINGTON PUBLIC SCHOOLS, Bloomington, Minnesota
Permision was granted to include material from two leaflets published by the Bloomington Independent School District #271 (Louis S. Cohen, Coordinator) entitled, *A Rationale in Working with Signed Numbers*, and *A Rationale in Working with Signed Numbers - Revisited*, and material taken from *Bloomington's School Mathematics Program*.

JOHN WILEY AND SONS, INC.
Ten Commandments for Teachers taken from George Polya's book, *Mathematical Discovery, Volume II*, copyright 1965 and published by John Wiley and Sons.

STATE DEPARTMENT OF EDUCATION, Santa Fe, New Mexico
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PHILOSOPHY

In the choices of life, present and future, that the individual may be called upon to make, the limiting factors should only rest upon the abilities, mental, physical and spiritual, of that individual and not upon the lack of a basic education. Within the limits of abilities man should have reasonable latitude for choices and not be limited by education or lack of it. An understanding of basic principles of mathematics is one of the keys to this education. Since there are only a few major strands that unify mathematics, mastery of these will make possible better choices in the life our students face. It is recognized that mastery of these major topics can be accomplished in many ways. However, it is felt that the specific ideas can be clearly established and defined with performance objectives. It is imperative that the teacher have clearly established the performance objectives for each student at each level of development and then use the methods necessary to attain these objectives.

An attempt has been made to identify a minimal set of performance objectives necessary for student achievement. Furthermore, many suggestions to gain this student achievement are included. Teachers are encouraged to select those suggestions that are the most appropriate in helping students gain an appreciation and mastery of mathematics.

GENERAL PURPOSE

THE IDAHO CURRICULUM GUIDE IN MATHEMATICS, GRADES K-8, WAS DESIGNED TO:

- -present a desired minimum of specific learning objectives from the field of mathematics for grades K-8.
- -present these learning objectives in terms of student behavioral or performance outcomes.
- -assist the teacher to view student growth in performance terms, where practical, rather than in terms of topics covered or in terms of teacher behavior.
- -provide knowledge of and encourage the use of alternative approaches for reaching the objectives detailed in the guide.
- -provide guidance so that the classroom teacher may supplement or occasionally replace methods or objectives suggested in published texts.
- -cause the teacher to prefer, select or devise classroom laboratory experiences to "text-only" experiences.
- -assist the teacher to recognize and plan for varied student abilities rather than treat all learners in identical ways.
- -better enable the teacher to recognize and plan for skills or concept developments which are prerequisites to new tasks.
- -provide the teacher with knowledge of the objectives of grade levels preceding and following the grade he is teaching.
- -enable the teacher to better select learning experiences, employing higher levels of cognitive involvements where possible.
- -enable the teacher to better evaluate learning outcomes.
- -enable the teacher to strengthen the clarity of his instructional goals.

MAJOR TOPICS

Major topics are those concepts and applications which pertain to an area of mathematics which students study. The content of this guide has been organized under five major topics:

-NUMBER AND OPERATIONS --Yellow*

-SETS, FUNCTIONS, RELATIONS, SYSTEMS AND LOGIC --Green*

-GEOMETRY --Pink*

-MEASUREMENT AND ESTIMATION --Blue*

-SELECTED TOPICS

A Scope and Sequence chart follows but the listings for each grade level should be regarded only as a suggested guide for introducing various topics.

*This color code is followed throughout the guide.

TEACHING AIDS

Teaching aids are those ideas, devices, games, and materials which introduce and illustrate those concepts and applications of the various topics. They should appeal to the intuition and encourage student involvement and discussion. Teaching aids offer opportunities for mental imagery. Students need a variety of experiences to keep the building of mathematical structure stimulating and strong. The teaching aids are offered as suggestions depending on the needs and abilities of the students.

BEHAVIORAL/PERFORMANCE OBJECTIVES

This curriculum guide, in addition to listing specific topics and particular teaching aids for each topic, describes specific objectives for each topic. These objectives have been written in terms of observable student behavior; and they clearly define for the student

(1) The desired behavior
(2) The conditions under which the performance should occur.

(3) Since behavioral objectives have to do with the observable actions of students, vague terminology such as "understand" and "know" is not used. Rather, words which describe student action such as "demonstrate", "state", and "write" are used. These actions of the student are ones by which the teacher may observe performance that indicates the student does "understand" a particular concept.

The behavioral objectives were written with the average student in mind. Therefore, it is felt that from 75% to 80% of the students in a typical class should be able to accomplish all the objectives of their particular grade level. The school staff should determine the degree of success that determines mastery and the specific student (such as slow, gifted, average) audience. Since many of the students may have accomplished some of the objectives of a particular grade level, a comprehensive program of pre-testing and post-testing can be utilized in conjunction with behavioral objectives to eliminate extra work for the student.

Sample behavioral objectives for the topic of order relations for natural numbers would be:

(1) Given two natural numbers such as 25 and 34, the student must be able to correctly order them as

$$25 < 34 \text{ and } 34 > 25$$

(2) Given three natural numbers such as 99, 100, 101, the student must be able to correctly order them from greater to lesser as

$$101 > 100 > 99$$

The "given" part establishes the circumstances under which the student is expected to perform, and the verb phrase "must be able to correctly order" indicates the student action which demonstrates successful accomplishment of the desired objective. In addition, the behavioral objective is easily translatable into test criteria by which the student's accomplishment may be measured.

Although behavioral objectives are not an absolute answer to educational problems, they do have the advantage of allowing the teacher to measure student accomplishment and thereby provide concrete criteria by which the student may proceed in the learning situation.

PROBLEM SOLVING

Problem solving is more than a major topic as defined in this curriculum guide. Problem solving is a part of everyone's daily life so every student needs to develop problem-solving skill. To attain this skill, we need to utilize problem solving behaviors such as observing, exploring, decision-making, organizing, recognizing, remembering, supplementing, regrouping, isolating, combining, diagramming, guessing, classifying, formulating, generalizing, verifying and applying. Teachers use these behaviors in the classroom every day and should emphasize them in teaching mathematics. Mathematics should help train students to think for a purpose. Therefore, the more a student practices and uses problem-solving behaviors, the better he will be able to grasp and solve any problem with which he is confronted, not only in mathematics but in every facet of his daily life.

The term problem is defined as any situation where one solves questions by means of something more than rote. What may be a problem to one student, may be merely an exercise to another, and a frustration to still another. If the capable student in the seventh grade is required to find the product of 45,987 x 37,478 with no particular purpose in mind but to give him drill, then this is simply an exercise. This is no problem for him in a normal sense. He may, however, make a problem out of it by trying to find a way out of the tedium involved. He either knows the necessary basic multiplication facts and the accompanying algorithm...or he doesn't. If he doesn't know his facts, and is not allowed to look them up on a table, he ends up frustrated. An example of a problem--

> A student brings 30¢ for his hot lunch. In how many ways may different coins be combined for a total of 30¢ for his lunch? Are there twice as many combinations if he brings 60¢?

The teacher should emphasize that some problems may be solved in a variety of ways.

Each person has his own style or way of solving problems. The results of previous problems solved in his day-to day living are constantly changing his approaches to problem solving. Mathematics offers a wide field of opportunity for experience in solving problems. Essential ingredients for solving problems are desire and determination. The more personally involved the student becomes, the more concepts and ideas he may retain for use in working out future problems.

It may help if a problem applies to the experiences of the student. He must understand the meaning of the problem and be able to identify available materials and methods to solve it. He must also discern relationships and be able to make appropriate choices in approaches to the problem. Analysis of a problem may involve, generally, in this order:

1. Concrete or manipulative devices, and laboratory work
2. Sketches
3. Hypotheses and guesses
4. Trials and testings.

Problem Solving . . .

Logic is essential in problem solving, although random trials sometimes lead to solutions. The discovery of a workable plan and a solution gives the student a feeling of achievement. The system of attacking a problem obtained from discovering a solution is often more important than the solution itself.

There are many other approaches to problem solving. One of these that fits many problems is the following:

- .First, observe and record data of the situation.
- .Formulate a hypothesis based on the data.
- .Predict from the hypothesis for additional data.
- .Test the prediction.
- .If necessary, correct the hypothesis from additional observation.

The foregoing method is called the Scientific Approach. This fits problems involving patterns and/or probability. More of this type of problem should be included in the curriculum.

There are many other types and techniques of problem solving. Some suggestions are listed below:

1. Give the student a set of numbers. Let him develop a story which uses the numbers. This might train him to think in terms of problems.

2. Ask the student to start thinking about problems connected with ordinary situations, such as:

   - .does his house have an odd number of doors
   - .how many times does he pass through each of them today, an even or odd number of times
   - .how many times through the door (or doors) of the room he is in now?

3. Have the class act out the problem. The following problem is a bit difficult to solve. However, in acting out the problem, the solution becomes quite apparent:

   A tramp finds a phony $10 bill, but is not aware that it is phony. He goes to a shoe store and purchases a $5 pair of shoes, giving the clerk the phony $10 he has found in payment. The clerk goes to the cash register and finds that he has a good $10 bill and four $1 bills in the till and cannot make change. He takes the phony bill to the shop next door and exchanges it for two $5 bills. He then returns, gives the tramp $5 change and puts the good $5 in his cash register. In a few minutes, after the bum has gone, the owner of the store next door, having discovered that the $10 bill is phony, brings it back and demands that it be replaced with a genuine $10 bill, which the shoe merchant does.

QUESTION: Just how much did the shoe merchant lose in this transaction?

The point of this is simply that the students acted out the procedures to find the answer and all were convinced of the correct answer.

4. Try the art of trial and error in solving problems. Many of the typical story problems in texts can be solved by trials until a solution is found. Perhaps the teacher may object to problem solving by this method because the student does not set up the problem with equations involving x's and y's. Certainly, the use of equations in solving problems is very important, when applicable. Many problems do not readily lend themselves to solution with equations and often the trial and error method is the only reasonably quick way available to the student, based upon his background. Consider, for example, this problem:

   Using only a 3-liter* bucket and a 5-liter* bucket, how is it possible to get precisely four liters* of liquid in the 5-liter* bucket?

   This is easily solved when actual pouring is done.

Trial and error problem solving is important. It requires the student to think, to organize his trials from easier to harder, to use previously learned experiences in successive trials, to check his solutions, and to summarize his solutions.

5. Vary the data to be used in problem solving. Give too much data by including data that is inappropriate to the problem. Students have been conditioned to expect to use all the data given, so do not develop discernment in determining what facts must be known to solve the problem. Try adding this sentence to any problem:

   You are given two 6-volt dry cells that weigh 4 pounds apiece.

   Another plan that should be followed is to give a problem with too little data. More of these types of problem-solving situations are found in real life than those in which the exact amount of data is available.

6. Offer problems that have many solutions, one solution, and no solutions.

   It should be learned that all problems do not have just one answer . . . or even an answer. Consider this easy problem:

   How many 6¢ and/or 10¢ postage stamps may I buy with 25¢ and receive no change?

   The answer is, obviously, none.

   For 80¢, how many 6¢ and 10¢ stamps may I buy without receiving change?

   Again, it can be seen that there are several possible correct solutions. But for 44¢, there is only one correct solution if there is to be no change.

---

* Liter and Meter may be spelled "Litre" and "Metre." However, this is according to the International System. In this guide, we will be consistent with the spelling "er." It is our assumption that it will be standardized in the U.S.

Problem Solving . . .

The role of the teacher is an important factor in problem solving.

The teacher can aid by presenting the problem so that the student can discover a pattern. The teacher should analyze trials that lead to incorrect solutions. The student may thereby learn some new concepts as well as learning from his mistakes. Computational ability is important in day-to-day living. Problem solving offers the most practical opportunity to develop proficiency in this skill.

A few suggestions are listed to help the teacher in attaining problem-solving success. They may serve as guidelines for an exploratory climate for problem solving.

1. Develop an enthusiastic attitude toward problem solving.
2. Make sure the statement of the problem is clear to the student.
3. Approach a topic from as many different points of view as possible.
4. Ask many leading questions.
    a. How will I know the solution works?
    b. Why doesn't it work"
    c. Is this the solution we want?
    d. Can this method be used for some other problem?
5. Give the student time to think and time to answer questions.
6. Don't be afraid to start over if the student becomes confused.
7. Capitalize on errors so that the student sees a need for checking.

Problem solving pervades every major topic in this curriculum guide. It is a rich opportunity to prepare students for effective living in our quantitatively-oriented society.

* * *

The "master" of problem-solving research, George Polya, cites these Ten Commandments for Teachers (Mathematical Discovery, p. 116, copyright 1965, Volume II):

TEN COMMANDMENTS FOR TEACHERS

1. Be interested in your subject.
2. Know your subject.
3. Know about the ways of learning: The best way to learn anything is to discover it by yourself.
4. Try to read the faces of your students, try to see their expectations and difficulties, put yourself in their place.
5. Give them not only information, but "know-how,"
6. Let them learn guessing.
7. Let them learn proving.
8. Look out for such features of the problem at hand as may be useful in solving the problems to come--try to disclose the general pattern that lies behind the present concrete situation.
9. Do not give away your whole secret at once--let the students guess before you tell it--let them find out by themselves as much as is feasible.
10. Suggest it, do not force it down their throats.

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SCOPE AND SEQUENCE

NUMBER AND OPERATIONS

	K	1	2	3	4	5	6	7	8
A. THE NATURAL NUMBERS - COUNTING AND NUMERATION									
1. Concepts related to counting	I	E	E	-	-	-	-	-	-
2. Counting the number in a set	I	E	E	-	-	-	-	-	-
3. Reading, recognizing, and writing numbers	I	E	E	-	-	-	-	-	-
4. Using ordinal names	I	E	E	E	-	-	-	-	-
5. Place value		I	E	E	E	E	-	-	-
6. Non-decimal numeration				I	E	-	-	-	-
7. Rounding and estimation				I	E	E	E	-	-
8. Primes and composites						I	E	E	E
9. Exponential notation						I	E	E	E
B. THE NATURAL NUMBERS - ADD, SUB, MULT, DIV.									
1. Addition and subtraction	I	E	E	E	E	-	-	-	-
2. Properties of the natural numbers		I	E	E	E	-	-	-	-
3. Multiplication and division			I	E	E	E	E	-	-
4. Properties of natural numbers			I	E	E	E	E	-	-
5. Greatest common factor - least common multiple									
C. THE RATIONAL NUMBERS									
1. Reading and writing the fractional numeral		I	E	E	E	-	-	-	-
2. Equivalent fractions and inequalities			I	E	E	E	E	E	-
3. Ordering fractions					I	E	E	E	E
4. Number pairs, ratio, and proportion						I	E	E	E
5. Decimal fractions (Comparison)						I	E	E	E

Code
I = Introduction
E = Extension
- = Maintain Concepts and Skills

NUMBER AND OPERATIONS CONT.

	K	1	2	3	4	5	6	7	8
D. THE RATIONAL NUMBERS - ADD, SUB, MULT, DIV.									
1. Addition and subtraction of fractions				I	E	E	E	E	E
2. Properties of addition and subtraction						I	E	E	E
3. Multiplication and division of fractions						I	E	E	E
4. Properties of multiplication and division						I	E	E	E
5. Add, Sub, Mult, Div. of decimals					I	E	E	E	E
E. THE INTEGERS									
1. Ordered pairs							I	E	E
2. Coordinate axes							I	E	E
3. The negative integers					I	E	E	-	-
4. Properties of naturals extended to integers								I	E
5. Absolute value									I
6. Addition and subtraction of integers					I	E	E	E	E
7. Multiplication and division of integers					I	E	E	E	E
F. THE IRRATIONAL NUMBERS									
1. The number line (Ordering)							I	E	E
2. Simplification and operations - radicals									I
3. Radical equations									

SETS, FUNCTIONS, RELATIONS, SYSTEMS, AND LOGIC

	K	1	2	3	4	5	6	7	8
A. SETS									
1. Collections	I	E	E	E	E	E	E	E	E
2. One-to-one Correspondence	I	E	E	E	E	E	E	E	E
3. One-to-many Correspondence			I	E	E	E	E	-	-
4. Equivalent and non-equivalent	I	E	E	E	E	E	E	E	E
a. Empty set	I	E	E	E	E	E	E	E	E
b. Subsets		I	E	E	E	E	E	E	E
5. Cardinal Numbers	I	E	E	E	E	E	E	-	-
6. Set Union and Intersection	I	E	E	E	E	E	E	E	E
7. Commutative, Associative, Distributive	I	E	E	E	E	E	E	E	E
8. Concept of Multiplication and Division			I	E	E	E	E	E	E
9. Set Symbolism			I	E	E	E	E	E	E
a. Set Equality					I	E	E	E	E
b. Finite and Infinite						I	E	E	E
10. Venn Diagrams						I	E	E	E
B. RELATIONS AND FUNCTIONS									
1. "What's My Rule"		I	E	E	E	E	E	E	E
2. Number Patterns			I	E	E	E	E	E	E
3. Cross products of sets, ordered pairs								I	E
4. Relations								I	E
5. Functions								I	E
6. Graphing of relations (including inequalities and functions)								I	E
7. Trigonometric Functions									I

CODE

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SETS, FUNCTIONS, RELATIONS, SYSTEMS, AND LOGIC CONT.

	K	1	2	3	4	5	6	7	8
C. MATHEMATICAL SENTENCES									
1. Reading and writing true, false & open sentences.		I	E	E	E	E	E	E	E
2. Number sentences, involving more than one operation with integers and need for punctuation.				I	E	E	E	E	E
3. Number sentences with fractions.						I	E	E	E
4. Sentences involving absolute value.									I
5. Sentences involving a second power of variable (quadratic equations).									
D. SYSTEMS AND LOGIC									
1. Probability								I	E
2. If and Only If									I
3. Truth Tables									
4. Conditional, contrapositive, converse, inverse, denial									
5. Computer-algorith, flow charts, language									
6. Informal Deduction									

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GEOMETRY

	K	1	2	3	4	5	6	7	8
A. NON-METRIC GEOMETRY									
1. Relative position of objects	I	E	-	-	-	-	-	-	-
2. Relative sizes of objects	I	E	-	-	-	-	-	-	-
3. Recognition of plane figures	I	E	E	-	-	-	-	-	-
4. Recognition of solid shapes	I	E	E	-	-	-	-	-	-
5. Points, line segments		I	E	-	-	-	-	-	-
6. Closed or open curves		I	E	-	-	-	-	-	-
7. Constructions, approximate and exact		I	E	E	E	E	E	-	-
8. Congruency			I	E	E	E	E	-	-
9. Recognition and description of plane figures				I	E	E	E	-	-
10. Recognition and description of solid figures				I	E	E	E	-	-
11. Set of points				I	E	-	-	-	-
12. Closed and open curves and regions				I	E	-	-	-	-
13. Symmetry				I	E	E	E	-	-
14. Similarity				I	E	E	-	-	-
15. Coordinate systems and graphs					I	E	-	-	-
16. Topology								I	E
17. Pythagorean property and special right triangles									I
18. Parallel and perpendicular									I
19. Cones, cylinders, spheres									I
20. Lines in relation to space									I
B. METRIC GEOMETRY									
1. Length (pre-formula inductive)		I	E	-	-	-	-	-	-
2. Area (pre-formula inductive)			I	E	E	-	-	-	-
3. Volume (pre-formula inductive)					I	E	E	-	-
4. Formulas for perimeters, areas, and volumes								I	E
5. Similar triangles and proportions								I	E

CODE
I = Introduction
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MEASUREMENTS AND ESTIMATION

	K	1	2	3	4	5	6	7	8
A. CONCEPT OF MEASUREMENT									
1. Process of Measuring		I	E	E	E	E	E	E	E
2. Arbitrary Selection of Unit		I	E	E	E	E	E	E	E
3. Approximation Nature of Measurement (Precision)				I	E	E	E	E	E
4. Concept of graphs			I	E	E	E	E	E	E
B. MEASURABLE QUANTITIES									
1. Time		I	E	E	E	E	E	E	E
2. Money	I	E	E	E	E	E	E	E	E
3. Linear Measurement	I	E	E	E	E	E	E	E	E
a. Perimeter					I	E	E	E	E
b. Circumference							I	E	E
4. Area				I	E	E	E	E	E
5. Volume				I	E	E	E	E	E
Liquid		I	E	E	E	E	E	E	E
6. Weight		I	E	E	E	E	E	E	E
7. Temperature		I	E	E	E	E	E	E	E
8. Metric System	I	E	E	E	E	E	E	E	E
9. Angle						I	E	E	E
10. Speed							I	E	E
11. Estimation with inequalities and in word problems							I	E	E
12. Graphs			I	E	E	E	E	E	E
13. Accuracy and Precision in Measurements							I	E	E
14. Indirect Measurement							I	E	E
15. Relative Error									I
16. Statistics					I	E	E	E	E
C. RENAMING MEASURES									
1. Comparison Units			I	E	E	E	E	E	E
2. Conversion Units			I	E	E	E	E	E	E
D. COMPUTATIONS WITH MEASURE				I	E	E	E	E	E

CODE
I = Introduction
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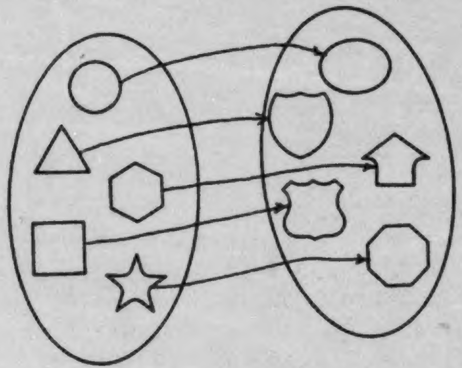
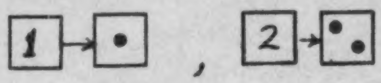
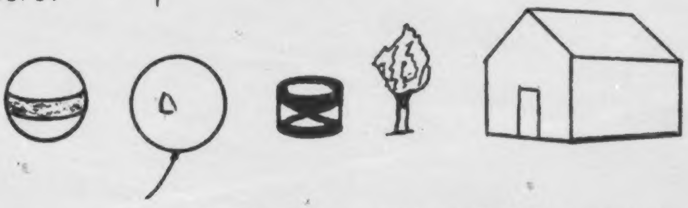
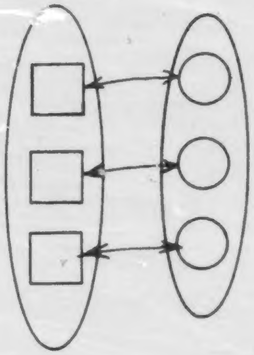
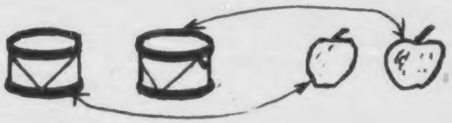
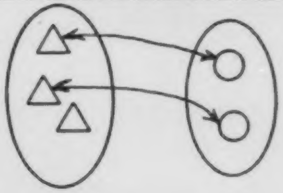
Major Topic	Behavioral Objectives	Teaching Aids
A. THE NATURAL NUMBERS-COUNTING AND NUMERATION		
1. Concepts Related to Counting	The student must be able to recite the first ten counting numbers in the proper order.	A large variety of experiences in counting many different types of objects is essential.
2. Counting the Number in a set.		
Collections	(a) Given a verbal description of a set, the student can distinguish between members of the set and things which are not members.	Ask students to name objects in the classroom that have a common characteristic such as green, round, square, flexible, flat, etc.
One-to-one correspondence	(b) Given two equivalent sets (objects or pictures), the student can demonstrate a one-to-one matching between members of the sets by constructing lines or by physically associating the objects.	In an activity such as using strips of yarn on a flannel board, have the students actually perform the activity rather than merely observe the teacher.
Equivalent and non-equivalent sets	(c) The student can distinguish between pairs of sets whose members can be matched one-to-one and pairs of sets whose members cannot be matched one-to-one.	
	(d) Given two non-equivalent sets, the student is able to identify the set which has fewer members.	
Cardinal Numbers	(e) The student can count the members of a set by simultaneously saying the number names (one to ten) in order and matching each number name to a member of the set.	

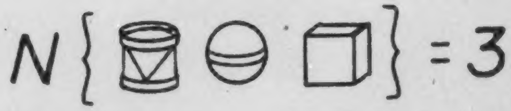
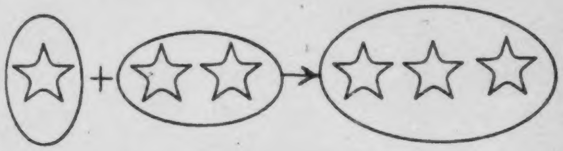
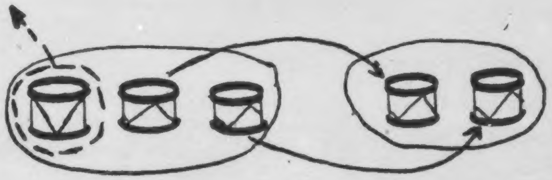
Major Topic	Behavioral Objectives	Teaching Aids
3. Reading, Recognizing, and writing Numbers 1-10	(a) Given a set with zero to ten members, the student can name how many objects are in the set by saying the number name or by selecting the correct numeral.	Have two sets of cards: one set containing corresponding numbers of objects. Let the students match the cards correctly and name the number. For example: 
	(b) Given a verbal or written numeral such as 7, the student can construct and identify sets containing seven members.	
4. Using Ordinal Names		
One-more pattern sets with cardinal numbers of 1-10	(a) Given ten non-equivalent sets, the student can arrange the sets in order. For Example: □, □ □, □ □ □, ...	Use the object cards described above and have the students arrange them in the correct order--both increasing and decreasing.
Order relations between numbers 0-10	(b) Given two numbers (verbal or written numerals) such as 7 and 3, the student can order them by saying:"7 is greater than 3" or "3 is less than 7".	Use the numeral cards described above. Give the student two or more cards--not necessarily in consecutive order--and have them arrange them in the proper order and state which is larger, etc.
Ordinal Numbers 1st through 3rd	(c) Given a sequence of objects, people, etc., the student can identify the 2nd object in the sequence.	

Major Topic	Behavioral Objectives	Teaching Aids
B. THE NATURAL NUMBERS-OPERATIONS		
1. Addition and Subtraction	(a) Given two disjoint sets of physical objects, the student can unite the sets and name the cardinal number of the new set thus formed.	Use the cards described in 3(a). Have the student select the correct numeral card to match the sum of two or more object cards.
	(b) Given two numbers such as 3 and 2, the students can design a simple experiment involving the union of two disjoint sets to determine and name the sum of the numbers.	Give the students a large number of opportunities to combine and determine the sum of a variety of sets of objects such as chips, students, books, sticks, etc.
Definition of subtraction (through sums of 10)	(c) Given a set of seven objects the student can identify and remove a subset with three members and name the cardinal number of the remaining subset.	NOTE: It is very important for the teacher to note the different approaches to subtraction demonstrated in (c) and (d) and to give the students sufficient experience in each type.
	(d) Given a set of five objects, the student can construct a set of nine objects and name how many more objects were needed to construct the second set.	

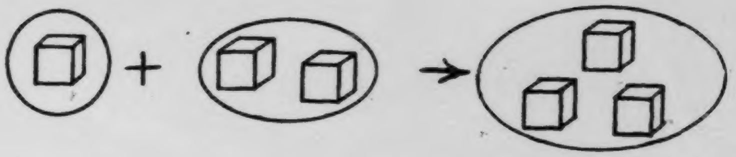
SETS, FUNCTIONS, RELATIONS, SYSTEMS, AND LOGIC KINDERGARTEN

Major Topic	Behavioral Objectives	Teaching Aids
A. SETS		
1. Collections	(a) Given a verbal description of a set, the student can distinguish between members of the set and things which are not members.	Use flannel board, collections of objects, and sets of pictures.  Display a set of objects such as this set of toys. (Balloon, Ball, Drums) Have the students select the objects that belong to the set.
2. One-to-one correspondence	Given two equivalent sets, the student can demonstrate a one-to-one matching between members of the sets by constructing lines between objects of two sets or by physically associating the objects.	 

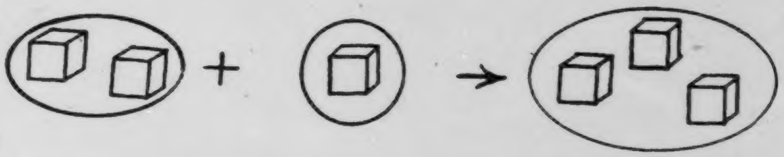
Major Topic	Behavioral Objectives	Teaching Aids
3. Equivalent and non-equivalent sets	(a) The student can distinguish between pairs of sets whose members can be matched one-to-one.	
	(b) Given two non-equivalent sets, the student is able to identify the set which has more members and the set which has fewer members.	
Empty set	(c) The student can describe verbally a set which has no members, such as the set of all live tigers in the classroom!	To develop the concept of the empty set, have paper bags filled with objects, and at least one bag containing no objects. Let the student draw an object from the bag. If he reaches into the empty bag he will discover that it contains no objects. This empty bag is like an empty set in that it contains no objects.
4. Cardinal Numbers 0-10	(a) The student can count the members of a set by simultaneously saying the number names (one to ten) in order and matching each number name to a unique member of the set.	Use popsicle sticks, counting men, blocks, balloons, etc.
One-more pattern	(b) Given ten non-equivalent sets, the student can arrange the sets in order.	X XX XXX XXXX etc. Use Cuisenaire Rods or Multibase Arithmetic Blocks. Sugar cubes glued together can be used, but save some to be eaten (without glue of course).

Major Topic	Behaviorial Objectives	Teaching Aids
Cardinal numbers	(c) Given a set containing zero to ten objects, the student can name how many objects are in the set by saying the number name or by selecting the correct numeral.	Use flannel board, collections of objects, and pictures of objects. 
5. Set Union Readiness for	(a) Given two sets, the student should be able to join the two sets and get a third set.	A set of one object joined to a set of two objects gives a set of three objects. 
	(b) Given a set of elements, the student should be able to remove a set and get a remaining set.	From a set of three objects remove a set containing one object which a set of two objects. 

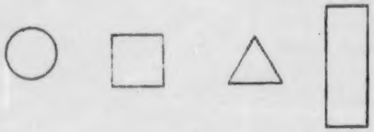
Major Topic	Behavioral Objectives	Teaching Aids
Commutative idea through sets	(c) The student should be able to demonstrate if you join a set, say, of one element to a set containing two elements, it is the same as joining a set of two elements with a set of one element.	

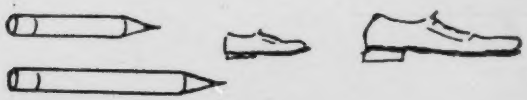
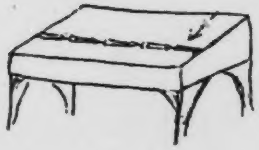


or



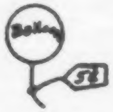
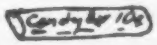
Major Topic	Behavioral Objectives	Teaching Aids
A. NON-METRIC ASPECTS		
1. Relative Position of Objects	Given an object, the child can say whether it is in front of, behind, below, on or above another object and... inside, outside of a boundary or place.	Large elastics, strings and ropes may be used to develop notions of inside, on, or outside. Games involving circles of children may contribute to understanding of inside, or outside. Boxes may also be used for ideas of inside, outside.
2. Relative sizes of Objects	Child can tell which objects are larger than, smaller than, or about the same size as other objects.	Children may move objects so as to compare them directly. Children may not yet utilize a separate standard for comparison and should probably not be taught to use a standard measure in this way at this grade level. Occasionally, objects may be placed on others to determine which is the larger of the two or may be filled with material such as water, sand, ping-pong balls, or blocks to determine which holds the most. Rather than teaching by telling, the teacher should encourage dialogue between the children as they offer notions and suggestions to verify their conclusions.
B. RECOGNITION OF PLANE FIGURES		
1. Plane Figures Circle Square Rectangle Triangle	(a) Given models of circles, squares, rectangles, and triangles (wire, papers, or flannel cut-outs, pencil or chalk outlines), the child can identify, name orally, and distinguish among these plane geometric figures.	Caution should be observed regarding the terms on or in. When the object is made from a wire, the concept of in or on is appropriate. However, a flat cut-out paper model needs a different interpretation for on and in than an outline model.

Major Topic	Behavioral Objectives	Teaching Aids
	(b) The children will use the terms inside, outside, and on as related to these figures formed by using framed outlines of the shapes. 	
C. RECOGNITION OF SOLID SHAPES 1. Solid Shapes Boxes Cubes rectangular prisms Balls spheres Cans cylinders	Given a set of objects, the child will name and sort them according to the categories at the left.	Children classify only according to the large categories, i.e., box, ball, cans.

Major Topic	Behavioral Objectives	Teaching Aids
A. CONCEPT OF MEASUREMENT		
1. Length Comparison	(a) Given a set of objects or pictures of objects, the student can compare them and identify and name the longest, shortest, tallest, and widest.	Use the flannel board for display of objects. Use pegboard and geoboard to mark segments and shapes for comparison. From a box containing objects of different sizes, the student can select the smallest or largest, etc.  Make comparisons of distance, size, and height, first using an arbitrary measure--such as pacing, etc.--later using an accepted scale such as the meterstick.  Desk is five pencils wide.

MEASUREMENT AND ESTIMATION

KINDERGARTEN

Major Topic	Behavioral Objectives	Teaching Aids
B. MEASUREMENT OF PHYSICAL PROPERTIES		
1. Money Dimes nickels pennies	The student recognizes pennies, nickels, and dimes by naming them.	Name the value of coins in actual situations, using milk money and play coins in dramatic play. Discuss equivalent values of coins. Play store by placing small objects on the display table. Each object should have a tag stating its money value. Have students take turns as storekeeper and customer.  

NUMBER AND OPERATIONS

GRADE ONE

Major Topic	Behavioral Objectives	Teaching Aids
A. THE NATURAL NUMBERS* COUNTING AND NUMERATION		
1. Concepts Related to Counting		
1-100	(a) The student can identify and name odd and even numbers.	Use the numeral cards described in Kindergarten Section A-3 (a) and have students separate them into appropriate piles.
1-100	(b) Given a number, the student can identify the numbers which immediately precede and follow it.	Give the student a numeral card and have him select the number immediately preceding and following from the cards displayed in the chalk tray.
2. Counting the Number in a Set		
Sets Collections	(a) Given a verbal description of a set, the student can distinguish between members of the set and things which are not members.	See discussion in Kindergarten Section A-2-(a).
	(b) Given two equivalent sets (objects or pictures) the student can demonstrate a one-to-one matching between members of the sets by constructing lines or by physically associating the objects.	See Kindergarten Section A-2-(b).
Equivalent & non-equivalent sets	(c) Given two sets (objects, pictures, verbal description), the student can identify them as equivalent or non-equivalent, and in non-equivalent, tell which set has more and which has fewer members.	

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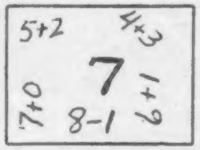
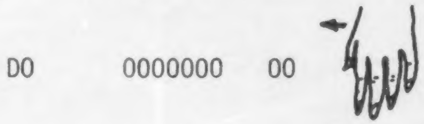
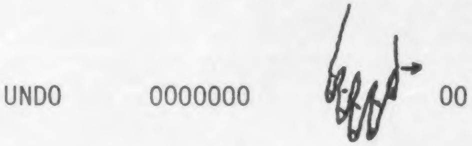
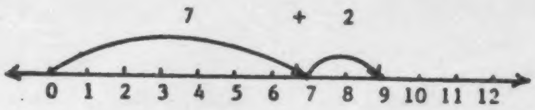
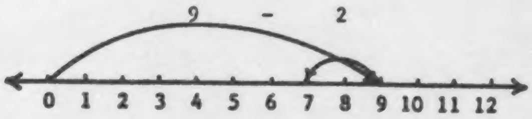
*Natural Numbers consist of the set $\{1, 2, 3, 4, 5, \ldots\}$

Major Topic	Behavioral Objectives	Teaching Aids
SETS (cont.) Empty Set	(d) The student can describe verbally a set which has no members, such as the set of all grandfathers in the classroom.	
Subsets	(e) Given a set, such as all students in the classroom, the student can identify a subset, such as the boys with brown eyes.	Encourage students to give examples of subsets of sets they can find in the classroom.
Cardinal Numbers 0-100	(f) Given a set, the student can count and name how many members are in the set.	
3. Reading, Recognizing and Writing Numbers		
Names for Numbers 0-20	(a) The student can identify and name different numerals for the same number, i.e. $5 = 5 + 1 = 3 + 2 = 7 - 2$, etc.	Student Activity: Use the numeral cards and cards showing the operation and equal signs to show different names for the same number. Visual Aids: Have large cards 75 cm x 100 cm with a numeral and some of the different names for that number printed on them. For example: 
Whole Numbers* 0-99	(b) The student can identify, name, read, and write numerals for whole numbers.	
	(c) The student can read and write number words one through ten.	

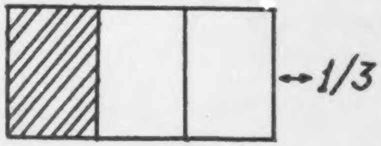
*Whole numbers are sometines called the counting numbers consisting of the set $\{0, 1, 2, 3, 4, . . .\}$

Major Topic	Behavioral Objectives	Teaching Aids
Place Value--two-digit numerals	(d) Given a numeral such as 73, the student can identify, name, and distinguish the numerals that are in the ones and tens places.	Use the numeral cards to have the student demonstrate the difference of 73 and 37.
4. Using Ordinal Names	(a) Given ten non-equivalent sets, the student can arrange the sets in order. For example: X XX XXX XXXX, etc.	Refer to Kindergarten Section A-4-(a).
Order relations	(b) Given two numbers such as 47 and 95, the student can order them by saying: "47 is less than 95" and by writing 47 95.	Use tongue depressors singly and in bundles of ten to establish the relative size of the two sets by matching.
Ordinal Numbers 1st.-10th	(c) Given a sequence of objects, events, etc. the student can identify the position in space or time of a particular object or event by saying: "This (object) is 4th" or "This (event) happened 2nd."	Arrange a row of objects in the room. Have students come up and point to the second, fifth, etc. The teacher will point to an object and have the student identify the 4th, 6th, etc.
5. Place Value		
Expanded notation	Given a numeral such as 54, the student can write the expanded numeral (50 + 4).	Use tongue depressors singly and in bundles of ten to demonstrate this.
B. THE NATURAL NUMBERS OPERATIONS		
1. Addition and Subtraction	(a) Given an addition problem such as 5 + 7=___, the student can design a simple experiment involving the union of two disjoint sets to determine and name the sum of the numbers.	Have available in the classroom a variety of materials to count such as chips, bottle caps, sticks, etc.

Major Topics	Behavioral Objectives	Teaching Aids
1. Addition and Subtraction (Cont.)	(b) Given a subtraction problem such as: 14-8= _____, the student can construct a set of 14 objects, remove a subset with 8 members and name the cardinal number of the remaining subset.	See Kindergarten Section B-1-(c). Also see note with Kindergarten Section B-1-(d).
	(c) Given a problem such as 6 + ___ = 15, the student can construct a set of 15 objects, remove a subset with six members, and name the cardinal number of the remaining subset.	
	(d) The student can demonstrate by using sets or a number line that subtracting 2 from 9 "undoes" adding 2 to 7.  7 + 2 = 9  9 - 2 = 7	A large number line should be available in the classroom.  

Major Topic		Behavioral Objectives	Teaching Aids
Basic facts Through sums of 10	(e)	Given an addition or subtraction combination such as 4 + 6 or 9 - 5, the student can *immediately name the sum or difference and use sets or a number line to prove his result.	Each student should have a number line available at his desk.
Through sums of 18	(f)	Given an addition or subtraction combination such as 7 + 8 or 16 - 9, the student can use sets or a number line to determine and name the sum or difference.	Teachers should constantly use both the tongue depressors and number line and materials (blocks, bottle caps, etc.) to assist the student to discover and reinforce these concepts. 14 4 -8; +7; ___ -6 = 4 14 - 8 = ___; 4 + 7 =___; 3 +___ = 12
	(g)	The student can identify and name sums differences, and missing addends in problems written in both horizontal and vertical notation.	For examples see above
	(h)	Given a problem such as 14 ___ 6 = 8, the student can identify and name the missing operational sign.	Use the numeral and operational symbol cards to provide opportunities for students to demonstrate various problems.
Algorithms: Column addition and subtraction	(i)	The student can name the sums and differences for problems such as 2 and 37 61 -13 +14	Use tongue depressors, Cuisenaire rods, Dienes blocks to provide an opportunity for the students to discover and reinforce these algorithms.

* Immediately is defined as 5 seconds or less.

Major Topic	Behavioral Objectives	Teaching Aids
2. Properties of the Natural Numbers		
Commutative Property of addition	(a) The student can demonstrate by using sets or a number line that: $7 + 4 = 11$ and $4 + 7 = 11$	Have the students use the tongue depressors or other aids and the number line.
Associative Property of addition	(b) The student can demonstrate by using sets or a number line that: $(7 + 2) + 8 = 9 + 8 = 17$ and $7 + (2 + 8) = 7 + 10 = 17$.	
Identity element for addition	(c) Given problems such as 0 + 14 = ____; 9 - ___ = 9; 7 - 7 = ___; and 32 + ___ = 32; the student can name the sums, differences, and missing addends and use sets or number line to prove his results.	
C. THE RATIONAL NUMBERS		
1. Reading and Writing the Fractional Numeral		
Rational numbers 1/2, 1/3, 1/4	(a) Given a model of one-third, the student can identify and name a numeral (fraction) for the rational number associated with the model. For example:  	Make felt cut-outs for use on the flannel board.

Major Topic	Behavioral Objectives	Teaching Aids
Rational Numbers one-half one-third one-fourth	(b) Given appropriate materials (paper regions Cuisenaire rods, etc.), the student can identify and construct models for 1/2, 1/3, and 1/4.	Paper folding would be very useful in demonstrating 1/2 and 1/4

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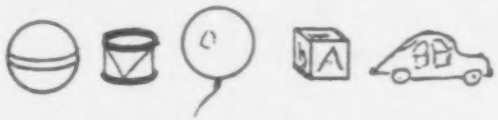
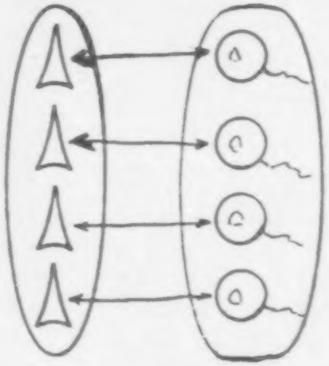
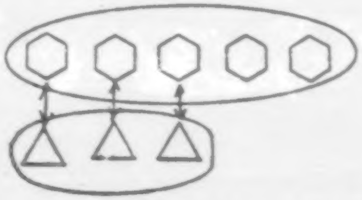
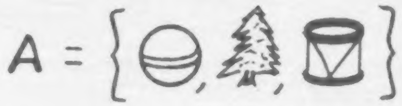
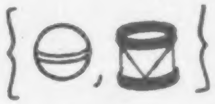
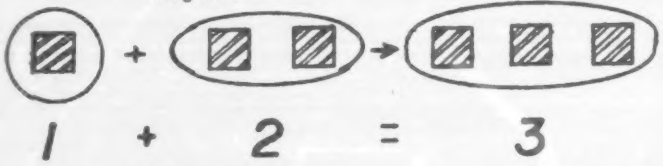
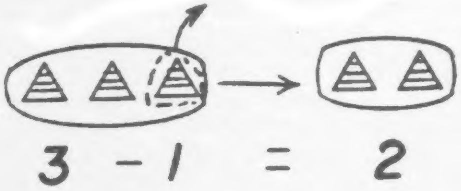
SETS, FUNCTIONS, RELATIONS, SYSTEMS, AND LOGIC

GRADE ONE

Major Topic	Behavioral Objectives	Teaching Aids
A. SETS		
1. Collections	Given a verbal description of a set, the student can distinguish between members of the set and things which are not members.	Use flannel board and collections of objects. Given a set of toys the student can select the toys that begin with B. 
2. One-to-one correspondence	Given two equivalent sets (objects or pictures), the student can demonstrate a one-to-one matching between members of the sets by constructing lines or by physically associating the objects.	Use collections of objects, pictures, etc. 
3. Equivalent & non-equivalent sets	(a) Given two sets (objects, pictures, verbal description), the student can identify them as equivalent or non-equivalent, and if non-equivalent, tell which set has more and which set has fewer members.	

Major Topic	Behavioral Objectives	Teaching Aids
One-more pattern	(b) Given ten non-equivalent sets, the student can arrange the sets in order.	X XX XXX XXXX XXXXX Use Cuisenaire Rods, Multibase Arithmetic Blocks, or sugar cubes glued together in "rods" to form this "staircase".
Empty set	(c) The student can describe verbally a set which has no members, such as the set of all the lions in the classroom!	To develop the concept of the empty set, have paper bags filled with objects, and at least one bag containing no objects. Let the student reach in a bag to remove an object. If he reaches into the empty bag he would discover that it contained no objects. This is like the empty set. It is a container with no objects in it.
Subsets	(d) Given a set, such as all students in the classroom, the student can identify a subset, such as the boys with brown eyes.	 The student can select a subset containing toys. 

Major Topic	Behavioral Objectives	Teaching Aids
4. Cardinal Numbers 0-100	Given a set, the student can count the members and name how many things are in the set.	Use popsicle sticks, counting men, blocks for the student to count. The student may not need this notation at this time. However, the teacher should be familiar with this. $N \{\square, \triangle, \bigcirc\} = 3$ The above is read: "The number of elements of the set is three.
5. Set Union Readiness for addition & subtraction	(a) Given two sets, the student should be able to join the two sets and get a third set.	A set of one object joined to a set of two objects gives a set of three objects. 
	(b) Given a set of elements, the student should be able to remove a set and get a remaining set.	Given a set of 3 objects, remove one object which leaves a set of two objects. 

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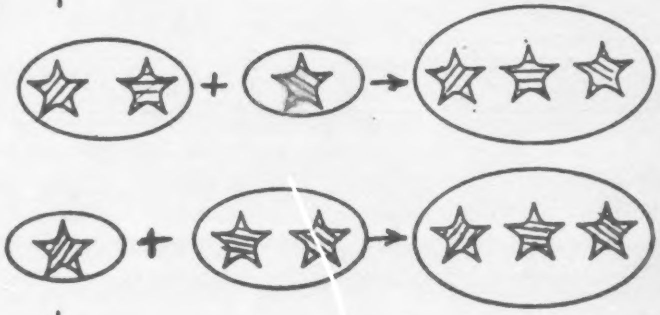
Major Topic	Behavioral Objectives	Teaching Aids

Commutative Idea

(c) The student should be able to demonstrate if you join a set, say, of one element to a set containing two elements, it is the same as joining a set of two elements with a set of one element.



B. RELATIONS & FUNCTIONS

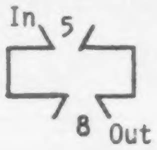
1. "What's My Rule?"

(a) Given the game situation described in the addenda under "What's My Rule?", the student will be able to determine the number that matches the question mark.

In	Out
1	2
5	6
7	8
4	?

In	Out
5	4
3	2
6	5
2	?

See addenda.
Use machines as in the following example:



In	Out
5	8
7	10
4	
1	

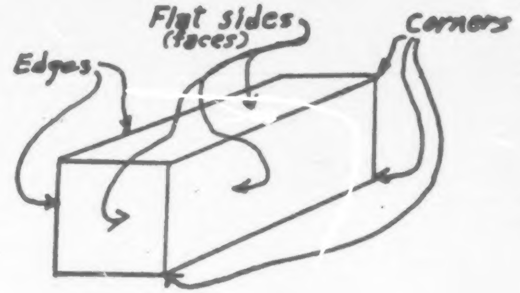
Mathematical Sentences

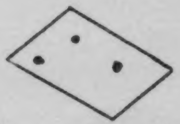
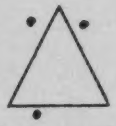
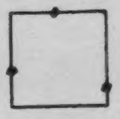
(b) Given several sentences, the student shall be able to distinguish and name open, true and false sentences.

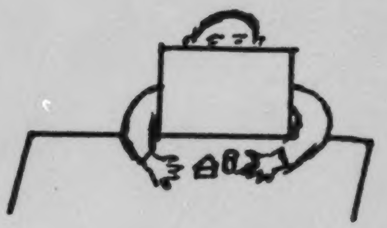
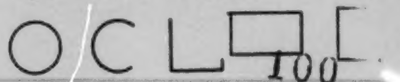
Open, $\square + 1 = 6$; true, $5 + 1 = 6$; false, $3 + 2 = 7$.

See addenda.

Major Topic	Behavioral Objectives	Teaching Aids
NON-METRIC ASPECTS		
A. POSITION AND SIZE OF OBJECTS		
1. Relative Position of Objects	Given an object, the child can say whether it is in front of, behind, below, on or above another object. and . . . inside, outside of a boundary or place and . . . on the side of (as to the left or to the right of another object).	Locations may be referred to within the classroom or in the larger environment as above and below the ceiling or floor, inside or outside the building on school property, in the city, etc.
2. Relative Sizes of Objects	The child can tell which objects are larger than, smaller than, or about the same size as other objects.	The resources or ways in which children can conclude these relationships may be expanded to include weighing or "determining mass" (gross measures only), filling containers with blocks, or children pacing distances, etc.
B. RECOGNITION OF PLANE FIGURES		
1. Plane Figures Circle Square Rectangle Triangle 4- and 5-sided figures	(a) Given models of circles, squares, rectangles, triangles, line segments, and a line, (wire, paper, or flannel cut-outs, pencil or chalk outlines), the child can identify, name, and distinguish among these geometric figures.	

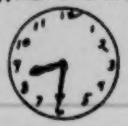
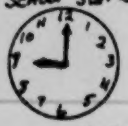
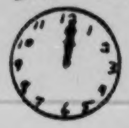
Major Topic	Behavioral Objectives	Teaching Aids
B. RECOGNITION OF PLANE FIGURES (continued)		
Circle	(b) Given rounded shapes, the child can select those which are or (are not) circles.	Objects should be clear examples of circles or not circles (circles, triangles, squares, etc.).
Line	(c) Given models or pictures of models with rounded and straight sides, the child can classify these into two groups and explain why he so classifies them.	
C. RECOGNITION OF SOLID SHAPES		
1. Solid Shapes Boxes cubes rectangular-prism Balls spheres Cans cylinders Cones Pyramids Triangular-prisms	(a) Given a set of objects, the child will name and sort them according to the categories at the left.	The rectangular prisms should be unlike cubes so that identification is readily made.
	(b) After some experience, the child can point to the edges, corners, flat sides of the objects and name the parts.	

Major Topic	Behavioral Objectives	Teaching Aids
2. Closed and Open Curves	Given a model or drawing, the child can identify squares, rectangles, triangles, and circles as closed curves and tell whether a point is inside, outside or on such a curve.	 Points inside  Points outside  Points on Caution needs to be observed regarding on or inside. If the model is of the border, only the inside, outside, and on are more readily evident.
3. Constructions-Approximate Circle Square Rectangle Triangle	(a) Given a pegboard and rubber bands, the child can construct a square, rectangle, and triangle. (b) The child can make rough pencil and/or chalk drawings (outlines) of circles, squares, rectangles and triangles.	The pupil should show how many varieties of each arrangement can be made. For a variety of problems, see Walker Geo Cards by Donald Cohen and the chapter on Geoboards in Notes on Mathematics in Primary Schools, Cambridge University Press.
METRIC ASPECTS		
A. PLANE FIGURES		
1. Length	(a) Given models of line segments of different lengths, the child can identify the longest and shortest line segments.	Use lines on chalkboard, pieces of string, or rope sections, sticks, etc.
	(b) Given a line marked with evenly-spaced marks, one of which is designated zero, the child can name numbers corresponding to the number of spaces from zero and also tell the number of spaces between any two marks.	Number of spaces or moves between marks should not be larger than cardinal values of numbers dealt with in other experiences. How many spaces does it take to move from zero to 3? From 3 to 7?

Major Topic	Behavioral Objectives	Teaching Aids
METRIC - PLANE FIGURES (Cont'd)		
Length	(c) Students will measure "distances", "heights", etc. using non-standard units of measure.	Non-standard units can be hand-spaces, pencil-lengths, length-of-foot, etc.
	(d) Students will use metersticks for measuring lengths--units to include centimeters and meters.	
2. Solid shapes "hidden" Ball, box and cylinder-shaped solids and cones	Given an object, they can "feel" but not "see"; the child can classify the object as to "shape."	Objects may be "inside" a bag or "behind" a screen. 
B. POINTS, LINE CONCEPTS		
1. Points, Line Segments	Given physical representation of points, line segments, corners and faces, the child can "distinguish between them" and "use their correct names."	The pupil may be asked to show which figures are "complete" and name them. "Incomplete forms" may be "incomplete" or "not circles" or "not squares." The idea of "incomplete" and "complete" adds the quality of "shape" to the concept of "open" and "closed." 

MEASUREMENT AND ESTIMATION

GRADE ONE

Major Topic	Behavioral Objectives	Teaching Aids
A. CONCEPTS OF MEASUREMENT		
1. Process of Measuring	Given an object with the property of length, such as a pencil, and a varied set of objects with which to measure the pencil, the student can select a suitable unit of length and measure the pencil by counting the numbers of units needed to match the length of the pencil.	Comparison should be made of distance, size, and height, first using an arbitrary measure--such as pacing, etc.--later using an accepted scale such as the meterstick. Introduce comparative terms, such as longer, high, low, etc. Children should be encouraged to "measure" familiar objects in their classroom, using units they invent themselves. If they use their feet or hands, questions will arise naturally, "Is everyone's foot the same size? Are some people's hands bigger than others?" Use such aids as Cuisenaire rods, Geo blocks, Dienes blocks, graph paper. Trace shoes or hands on graph paper and discuss what is meant by "bigger."
B. MEASUREMENT OF PHYSICAL PROPERTIES		
1. Time Clock Hour, half-hour	(a) Given a clock, or clock face, the student can tell time on the hour, and half-hour.	Time to catch bus  School starts  Lunch time 
Calendar Day, Week, Month	(b) The student can tell the name of the month and the day of the week.	

Major Topic

B-1(b) Measurement of Physical Properties (Continued)

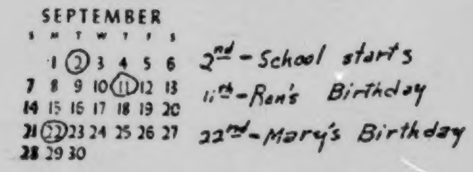
Behavioral Objectives

Teaching Aids

Children can be led to an awareness that the clock measures the time we spend on various activities: It tells us when to go out for the school bus, when to start our class, when to clean up, when to have our snack, etc. The concepts of early and late can be taught.

Time, as expressed by the passage of days, can be measured by the use of the calendar. A weekly time line calendar simplifies this understanding. "Counting down" the number of days to a holiday on a chain or button time line helps fix the concept of days passing and days counted. (A link in the chain or a button on a line is removed for each day.)

Allow children to put their birthdays on the calendar. Review events of the year and mark them on the calendar.



SEPTEMBER

S	M	T	W	T	F	S
	1	2	3	4	5	6
7	8	9	10	11	12	13
14	15	16	17	18	19	20
21	22	23	24	25	26	27
28	29	30				

2nd - School starts
11th - Ron's Birthday
22nd - Mary's Birthday

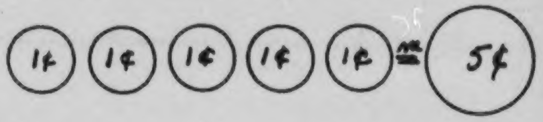
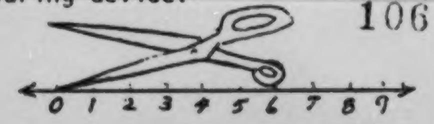
Make up number stories about the date:

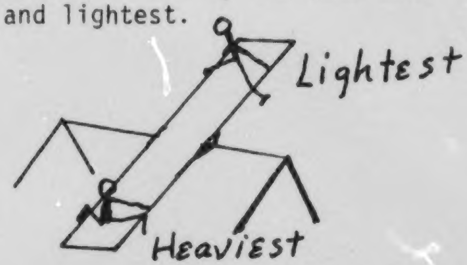
How many days between Ron's and Mary's Birthdays?

How many days to the end of the month from today? etc.

MEASUREMENT AND ESTIMATION

GRADE ONE

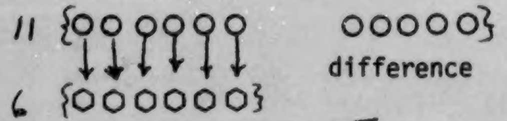
Major Topic	Behavioral Objectives	Teaching Aids
2. Money Penny, Nickel, Dime	(a) The student can identify and name pennies, nickels, and dimes and state the value, in cents, of the coins. (b) Given a set of pennies and/or nickels, the student can determine the name and total value, in cents, of the coins.	 Use names and values of coins in actual situations. Use milk money and play coins, in dramatic play. Discuss equivalent values of coins. Play the game: How Much Money Have I. See addenda.
3. Linear Measurement Length Ruler Centimeter Meter	The student can measure objects or pictures of objects to the nearest whole unit using both arbitrary units and ruler.	Have children measure objects using rulers that are marked in centimeters then measure the same object using a pencil. Let them decide which is the better object to use as a measuring device. For small objects use a ruler marked in centimeters. For larger objects use a tape marked in centimeters and meters. Have students draw lines using their rulers according to the teacher's direction or directions given by fellow classmates. The number line can be used as a measuring device. 

Major Topic	Behavioral Objectives	Teaching Aids
4. Volume Liquid measure 250 ml = 1/4 liter 500 ml = 1/2 liter 1000 ml = 1 liter ml is an abbreviation for milliliter	Based on experience with 250 ml, 500 ml, and liters, the student can compare and order these liquid measures. For example: 250 ml is (more, less) than 1 liter	Have 250 ml, 125 ml, 500 ml, liter, 4 liter containers near sink for children to do their own experiments. Questions for reinforcement of concepts taught: 1. Which is better to fill an aquarium, a liter or a 500 ml measuring pitcher? To fill a small fish bowl? 2. Mother has 1500 ml of juice: How many children can she serve if she gives each 250 ml of juice?
5. Weight -- Mass	Given a set of objects, the student can compare them and identify and name the heaviest and lightest.  	Make comparisons of objects to find out which is lighter and which is heavier. Are larger objects also heavier object? Introduce kilogram and grams as the units for measuring weight or mass. Use simple balance and scale to illustrate comparisons. Children can also estimate lighter and heavier by sight or by holding the two objects to be compared. See addenda.
6. Temperature Thermometer degree Celsius	(a) Based on experiences with reading scales on a thermometer, the student can record temperatures inside and outside the classroom and answer questions such as: Is it warmer inside or outside the classroom?	Room and outdoor thermometers can be used to make comparisons. Discuss body temperature, temperature as related to illness, cold-blooded animals in study of reptiles. Use cardboard thermometer. See addenda.

Major Topic	Behavioral Objectives	Teaching Aids
A. THE NATURAL NUMBERS-COUNTING & NUMERATION		
1. Concepts Related To Counting		
1 - 1000	(a) Given a number, the student can identify the numbers which precede and follow it.	Refer to Kindergarten Section A-1-(a)
1 - 1000	(b) The student can identify and name odd and even numbers.	
Order relations 1 - 1000	(c) Given a set of whole numbers, the student can write them in order from least to greatest and vice versa.	
2. Counting the Number in a Set		
Collections	(a) Given a verbal description of a set, the student can distinguish between members of the set and things which are not members.	Refer to Kindergarten Section A-2-(a)
One-to-one correspondence	(b) Given two equivalent sets (objects or pictures), the student can demonstrate a one-to-one matching between members of the sets by constructing lines or by physically associating the objects.	
One-to-many correspondence	(c) The student can describe familiar physical situations in which a one-to-many matching occurs.	For example: One hand to five fingers Two wheels to one bicycle One teacher to many students Have students give other examples.

Major Topic	Behavioral Objectives	Teaching Aids
Equivalent & non-equivalent sets	(d) Given two sets (objects, pictures, verbal description), the student can identify them as equivalent or non-equivalent, and in non-equivalent, tell which set has more and which fewer members.	Refer to Kindergarten Section A-2-(c)
Empty Set	(e) The student can describe verbally a set which has no members, such as the set of children in the classroom with green hair.	
Sub-sets	(f) Given a set, such as all students in the classroom, the student can identify a sub-set, such as the girls with blonde hair.	Refer to Grade One Section A-2-(e)
Cardinal Numbers 0 - 1000	(g) The student can group the members of a given set by ones, twos, fives and tens and determine by counting how many members are in the set.	Have students use tongue depressors, Cuisenaire rods, multibase arithmetic blocks, etc. for grouping into sets of twos, fives and tens and then count the total number of objects by twos, fives and tens.
3. Reading, Recognizing & Writing Numbers	(a) The student can identify, name, read and write many different names for the same number.	Have students regroup a given number of sticks to demonstrate the different names for the same number. For example: 123 = 100 + 20 + 3 = 90 + 20 + 13 = 130 -7, etc.

Major Topic	Behavioral Objectives	Teaching Aids
Whole Numbers 0 - 999	(b) The Student can identify, name, read, and write numerals for whole numbers.	
	(c) The student can read and write number words through twenty.	
4. Order Relations <,=,>,≠	(a) Given a mathematical sentence such as 15 + 12 ____ 30, the student can write the missing relation symbol.	Refer to Grade One Section A-4-(b)
First through Twentieth	(b) Given a set of objects, the student can correctly relate the ordinal numbers with the objects; i.e. this is the fifth block, etc.	
5. Place Value		
3-digit numerals	(a) Given a numeral such as 307, the student can identify, name and distinguish the numerals that are in the ones, tens, and hundreds places.	Have three sets of numeral cards 0 through 9. Many activities, games and relay races can be devised to have students demonstrate the difference between 305, 503, and 530 by placing the cards in the chalk tray or using an abacus.
Expanded Notation	(b) Given a numeral such as 586, the student can write the expanded numeral (500 + 80 + 6).	Refer to Grade One, Section A-5-(a)

Major Topic	Behavioral Objectives	Teaching Aids
B. NATURAL NUMBERS-OPERATIONS		
1. Addition & Subtraction Definition	(a) Given an addition or subtraction problem such as 5 + 7 = ____ or 14 - 8 = ____, the student can design a simple experiment involving sets or a number line to determine and name the sum or difference of the numbers.	Refer to Grade One, Section B-1 (a)
	(b) Given two sets, the student can compare the sets by matching the members and write a subtraction equation to express the difference between the cardinal numbers of the sets.	For example:  $11-6 = 5$
Inverse	(c) Given a set model, the student can write two addition and two subtraction equations to describe the physical situation.	For example: DO $6 + 7 = 13$ $7 + 6 = 13$ UNDO $13 - 7 = 6$ $13 - 6 = 7$

Major Topic	Behavioral Objectives	Teaching Aids

Basic Facts through sums of 18

(d) Given an addition or subtraction combination such as 7 + 8 or 16 - 9, the student can *immediately name the sum or difference and use sets or a number line to prove his result.

(e) The student can identify and name sums, differences, missing addends, and missing operational signs in problems written in both horizontal and vertical notation.

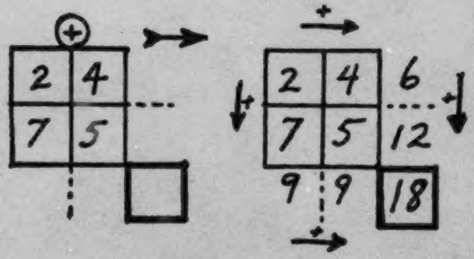
For example:

$\begin{array}{r} 14 \\ \underline{-8} \end{array}$; $\begin{array}{r} 9 \\ \underline{+\ 7} \end{array}$; $16 - 8 = \square$; $\begin{array}{r} 8 \\ \underline{+\square} \\ 10 \end{array}$; $\square - 6 = 4$;

$4 + 7 = \square$; $3 + \square = 12$;

$14 \bigcirc 6 = 8$; $9 \bigcirc 3 = 12$.

Cross Number Puzzles can be used for patterns and drill, for example:



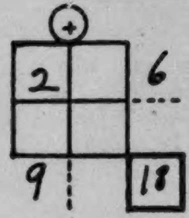
Refer to Grade One, Section B-1-(e)

Algorithms**
Column addition & subtraction (3-digit numerals) without regrouping

(f) The student can name the sums, differences, and missing digits for problems such as:

$\begin{array}{r} 141 \\ 22 \\ \underline{+413} \end{array}$; $\begin{array}{r} 247 \\ \underline{-123} \end{array}$; $\begin{array}{r} 2\square7 \\ \underline{+\ 1\ 5\square} \\ 9\ 8 \end{array}$; $\begin{array}{rrrr} & 5 & 6 & \square \\ - & \square & 2 & 1 \\ \hline & 1 & \square & 7 \end{array}$

Try this Cross Number Puzzle in addition to the one above:



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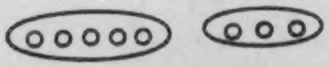
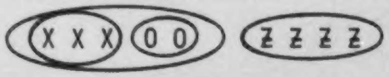
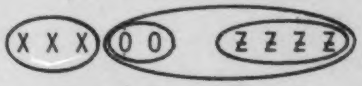
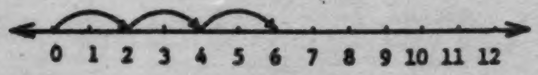
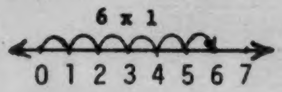
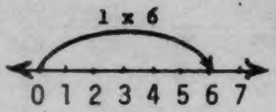
*Immediately is defined as 5 seconds or less.
**Algorithm simply means method or system for an operation.

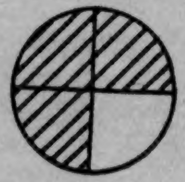
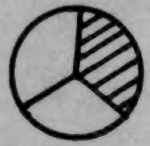
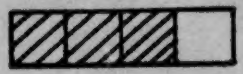
Major Topic	Behavioral Objectives	Teaching Aids
Column addition & subtraction (2-digit numerals) with regrouping.	(g) The student can name the sums and differences for problems. $\begin{array}{r} 34 \\ \underline{+58} \end{array}$ $\begin{array}{r} 52 \\ \underline{-35} \end{array}$	It is important for students to have a large amount of practice using tongue depressors or other materials to solve problems in addition and subtraction that require regrouping. It is desirable for as many students as possible to discover the algorithm. Note: Any of the following algorithms may be used. $\begin{array}{r} 34 \\ \underline{+58} \end{array}$ or $\begin{array}{r} \overset{1}{3}4 \\ \underline{+58} \\ 92 \end{array}$ or $\begin{array}{r} 34 \\ \underline{+58} \end{array} = \begin{array}{r} 30 + 4 \\ \underline{50 + 8} \end{array} = \begin{array}{r} 12 \\ \underline{+80} \end{array}$ $\begin{array}{r} \overset{4}{\cancel{5}}\overset{12}{\cancel{2}} \\ \underline{-35} \end{array} = \begin{array}{r} 50 + 2 = 40 + 12 \\ \underline{(30 + 5) = (30 + 5)} \\ 10 + 7 = 17 \end{array}$

Major Topic	Behavioral Objectives	Teaching Aids
2. Properties Commutative property of addition	(a) Given a set model, the student can write two addition equations to describe the physical situation.	For example:  5 + 3 = 8 3 + 5 = 8 This property can be demonstrated by drawing similar figures on a transparency and rotating the transparency by 180°.
	(b) The student can solve equations such as 2 + 6 = 6 + ___ & 4 + ___ = ___ + 4	
Associative property of addition	(c) Given a set model, the student can write an addition equation using parentheses to describe the physical situation. For example:  (3 + 2) + 4 = 9  3 + (2 + 4) = 9 .	The students can demonstrate this property by using the tongue depressors and some yarn to enclose the groups as the parentheseses enclose the numerals. It could also be demonstrated by using felt figures and yarn on the flannel board.
	(d) The student can solve equations such as (2 + 3) + 4 = ___ + 4 = ___ (7 + 4) + 2 = ___ + (4 + 2) and 3 + (9 + 1) = ___. (3 + 2) + 4 = 9	

Major Topic		Behavioral Objectives	Teaching Aids
Associative Property of addition (cont.)	(e)	The student can demonstrate his understanding of the regrouping principle of addition by solving equations such as $7 + 8 = 10 + \_\_\_ = \_\_\_$	
Identity element for addition	(f)	The student can solve equations such as $0 + 14 = \_\_\_;\ 9 - \_\_\_ = 9;$ $7 - 7 = \_\_\_;\ 32 + \_\_\_ = 32;$ $27 + \_\_\_ = 27 - \_\_\_;$ and $(2 + 0) + 9 = \_\_\_.$	
3. Multiplication & Division	(a)	Given several equivalent disjoint sets, the student can unite the sets and name the cardinal number of the new set thus formed.	
	(b)	Given a multiplication problem such as $4 \times 3 = \_\_\_$, the student can design a simple experiment involving the union of four equivalent disjoint sets each with three members to determine and name the product of the numbers.	For example: (X X X) (X X X) (X X X) (X X X) Refer to addenda on multiplication
	(c)	The student can solve equations such as $2 \times 3 = \_\_\_ + 3$ and $5 + 5 + 5 + 5 = \_\_\_ \times 5$ and use sets to prove his results.	

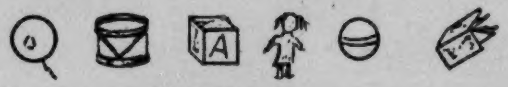
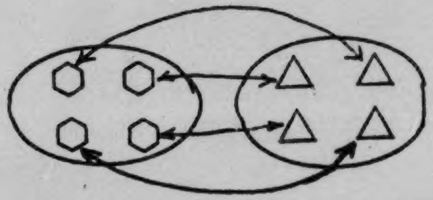
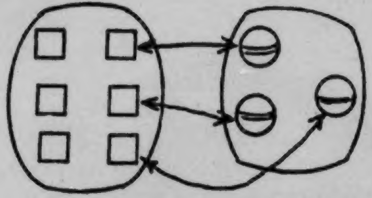
Major Topic	Behavioral Objectives	Teaching Aids
3. Definition of division (1-digit factors)	(d) Given a set of 15 objects, the student can partition the set into equivalent disjoint subsets each having three members and name how many subsets can be formed.	Approach division as the inverse of multiplication take 15 tongue depressors and determine how many groups of three can be formed.
Basic facts Through products of 45 (1-digit factors)	(3) Given a multiplication combination such as 3 x 2, the student can use sets or a number line to determine and name the product.	Refer to addenda on multiplication 
4. Properties on the Natural Numbers		
Commutative property of multiplication	(a) The student can demonstrate by using sets or a number line that 3 x 4 = 12 and 4 x 3 = 12	Use jumps on a number line. Arrange same dots on a transparency as follows: . . . . By rotating the transparency 90° on an overhead projector, it can be clearly demonstrated that 3 x 4 gives the same result as 4 x 3.

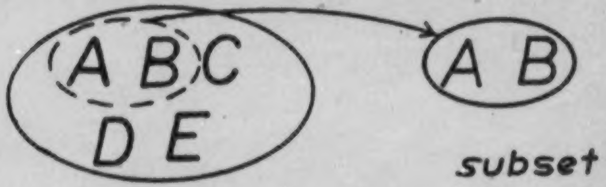
Major Topics	Behavioral Objectives	Teaching Aids
Identity element for multiplication	(b) The student can demonstrate by using sets or a number line that 1 x 6 = 6 and 6 x 1 = 6	6 sets of 1 X X X X X X 6 x 1 = 6  1 set of 6 X X X X X X 1 x 6 = 6 
Multiplicative Property of 0	(c) The student can solve equations such as 5 x 0 = ___ and 0 x 7 = ___.	

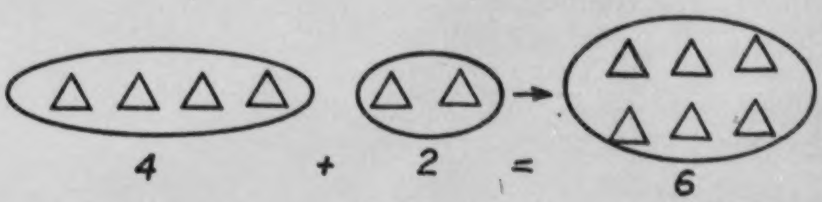
Major Topics	Behavioral Objectives	Teaching Aids
C. THE RATIONAL NUMBERS 1. Reading & writing the fractional number		
Rational Numbers 1/2, 2/2, 1/3, 2/3, 3/3, 1/4, 2/4, 3/4, 4/4,...	(a) Given a model of 3/4's, the student can identify, name, read, and write a numeral (fraction) for the rational number associated with the model.	For example: 3/4 
Rational Numbers Halves Thirds Fourths	(b) Given models of 1/3, 2/3, 3/3, etc., the student can identify and name the rational numbers associated with the models.	1/3   3/4

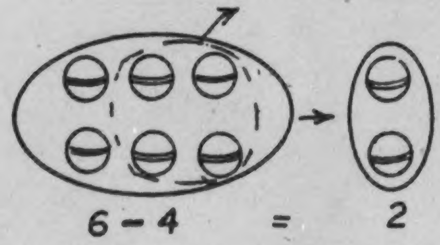
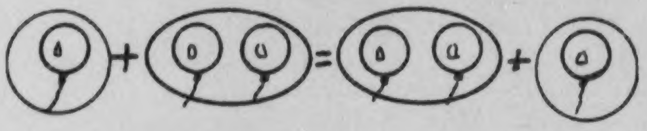
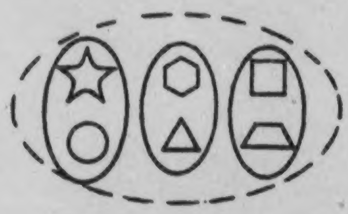
SETS, FUNCTIONS, RELATIONS, SYSTEMS, AND LOGIC

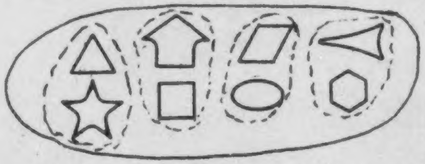
GRADE TWO

Major Topic	Behavioral Objectives	Teaching Aids
A. SETS		
1. Collections	Given a verbal description of a set, the student can distinguish between members of the set and things which are not members.	Use flannel board, and collections of objects. Given a set of toys the student can select the toys that begin with the letter "d" 
2. One-to-one correspondence	Given two equivalent sets (objects or pictures), the student can demonstrate a one-to-one matching between members of the sets by constructing lines or by physically associating objects.	Use flannel board with collections of objects and collections of pictures.
3. One-to-many correspondence	The student can describe familiar physical situations in which a one-to-many matching occurs. For example: one hand to five fingers, or two wheels to one bicycle.	
4. Equivalent & non-equivalent sets	(a) Given two sets (objects, pictures, verbal description), the student can identify them as equivalent or non-equivalent, and if non-equivalent, tell which set has more and which set has fewer members.	

Major Topic	Behavioral Objectives	Teaching Aids
Empty Set	(b) The student can describe verbally a set which has no members, such as the set of all live tigers in the classroom.	To develop the concept of the empty set, have paper bags or boxes containing objects and at least one bag or box containing no objects. Let the student draw an object from a bag or box. If he reached in the empty bag or box he would discover that it contained no objects. This is similar to the empty set, a container with no objects in it.
Subsets	(c) Given a set, such as all students in the classroom, the student can identify a subset, such as the girls with red hair.	Use flannel board to display objects. Have a collection of objects available to make sets. 
5. Cardinal Numbers 0-1000	The student can group the members of a given set by ones, twos, fives, and tens, and determine by counting how many members are in the set.	Use popsicle sticks, counting men, blocks, and bottle caps in egg cartons.
6. Set Union	(a) Given two sets, the student should be able to join the two sets and get a third set.	**Example:** A set of two objects joined to a set of four objects gives a set of six objects.



Major Topic	Behavioral Objectives	Teaching Aids
Set Union	(b) Given a set of elements, the student should be able to remove a set and get a remaining set.	Example: Take a set of six objects. Remove a set of 4 objects. This leaves a set of two objects. 
Commutative idea	(c) The student should be able to demonstrate if you join a set, say, of one element to a set containing two elements with a set of one element.	Use flannel board, Cuisenaire Rods, and collections of objects. 
Concept of Multiplication	(d) The student can describe set union as multiplication as in the example: Three two's or 3 x 2 =	Use flannel board and collections of objects. 

Major Topic	Behavioral Objectives	Teaching Aids
Set Partitioning	(e) The student shall be able to describe set partitioning as division. For example: 8 - 4 = □	Secure collections of objects and pictures that can be divided into sets and subsets. 
Preparation for Subtraction	(f) The student shall be able to compare two non-equivalent sets and tell how many more elements would have to be added to the smaller set to have as many elements as the larger set.	Use collections of objects, and collections of pictures containing different number of objects.
7. Set Symbolism	(a) Given a description of Set A and a larger set (Set B) containing Set A, the student can select Set A from the larger set.	Set A = {○, □, △, ☆} and the larger set Set B = {☾, ○, □, △, ▱, ☆}
N (a) =	(b) The student shall be able to associate a numeral with a set that names the number of elements in the set either by using the set notation or simply naming the number of elements of the set.	For example: where Set B = {□, ○, △, ♡} then the n(B) = n{□ ○ △ ♡} =4 or the number of elements in Set B is 4.

SETS, FUNCTIONS, RELATIONS, SYSTEMS, AND LOGIC

GRADE TWO

Major Topic	Behavioral Objectives	Teaching Aids
B. RELATIONS AND FUNCTIONS		
1. "What's My Rule?"	Given the game situation described in the addenda, the student will be able to determine the number that matches the question mark.	See addenda.

In	Out
3	6
1	2
5	10
4	?

In	Out
3	5
1	3
5	7
4	?

In	Out
(1,2)	3
(2,4)	6
(5,1)	6
(7,2)	?

In	Out
5	15
8	18
9	-
11	-

Major Topic	Behavioral Objectives	Teaching Aids
2. Number Patterns	The student shall be able to complete the next two elements of the following sets.	See addenda.

$A = \{1, 3, 5, \_\_, \_\_, \ldots\}$

$B = \{2, 5, 8, 11, \_\_, \_\_, \ldots\}$

Major Topic	Behavioral Objectives	Teaching Aids
C. MATHEMATICAL SENTENCES		
1. Open, true, & false sentences	(a) The student shall be able to classify the following types of sentences as open, true, or false, and answer or correct the open and false sentences. $5 - \boxed{} = 3$ $3 + 2 = n$ $5 \times 3 = 17$	Open $7-\boxed{} = 4; 3 + 5 = n$ True $5 + 3 = 8; 7 + 1 = 8$ False $6 \times 2 = 11; 6 - 2 = 5$
	(b) The student shall be able to use the "is not equal" sign.	$5 + 2 \neq 9$ $6 - 4 \neq 3$

GEOMETRY

GRADE TWO

Major Topic	Behavioral Objectives	Teaching Aids
A. RECOGNITION OF PLANE FIGURES		
1. Plane Figures circle square rectangle triangle 4-and-5-sided figures Line Segment Line (number line)	Given models of circles, squares, rectangles, triangles, line segments, and a number line (wire, paper, or flannel cut-outs, pencil or chalk outlines), the child can identify, name, and distinguish among these plane geometric figures.	The teacher should be sure that two things happen in the classroom with regard to recognition of plane and solid figures. From a given set of objects containing the shapes listed to the left, the child should name any object contained in the set that the teacher names.
B. RECOGNITION OF SOLID SHAPES		
1. Solid Shapes Boxes cubes rectangle prism Balls spheres Cans cylinders Cones Pyramid triangular prism	(a) Given models of cylinders and cones (wood or plastic solids, rolled paper, etc.), the child can identify, name, and distinguish between these space figures.	See above.
Ball, box & cylinder-shaped Solids & Cones.	(b) Given a set containing balls, boxes, and cylindrical-shaped objects, the child can group them according to shape and tell why he grouped them.	Child is to sort according to common characteristics (large, small, rounded, square, triangular, etc.)

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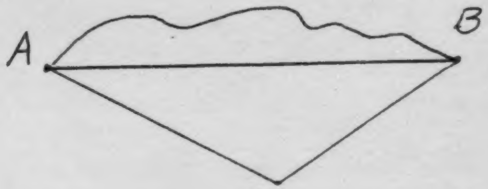
Major Topic	Behavioral Objectives	Teaching Aids
C. POINTS, LINE CONCEPTS		
1. Points, line Segments	(a) The student can name and identify line segment or curve as a set of points.	A point is an idea which can be thought of as an exact location. The concept of point is undefined in geometry. A point, like a number, is an idea existing in the mind. A point has no width or thickness, hence, does not occupy space.
	(b) The pupil can identify a straight line as a set of points with no beginning and no end.	While most of our models are of segments, the pupil may be led to think of a line as having no end. Have two children with balls of connected string hold the line tight and step backwards. This would illustrate a line as unending.
2. Closed & Open Curves	(a) The student can identify a simple curve (in a plane) as one that does not cross itself.	
	(b) The pupil can name and identify closed and open simple curves.	There may be many paths between two points. Some of the paths between points A and B are shown in the following drawing: 

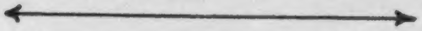
Major Topic

Behavioral Objectives

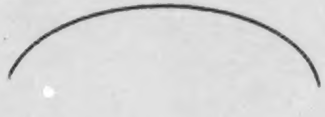
Teaching Aids

2. Closed & Open Curves (cont.)

In geometry, paths are called curves. Curves are sets of points. A line is a particular curve (set of points) that corresponds to our idea of a straight line. Drawings for straight lines may be made by using a straight edge. A model of a straight line is shown in the following diagram, where the arrows indicate that a line goes on and on indefinitely.

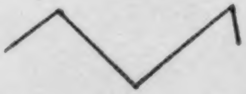


When referring to straight paths, use the word line; when referring to other paths, use curve or paths, as appropriate. Some models for representing paths are pictured in the following:

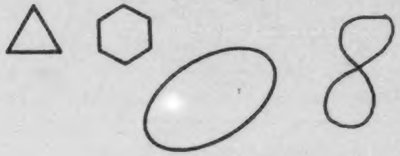
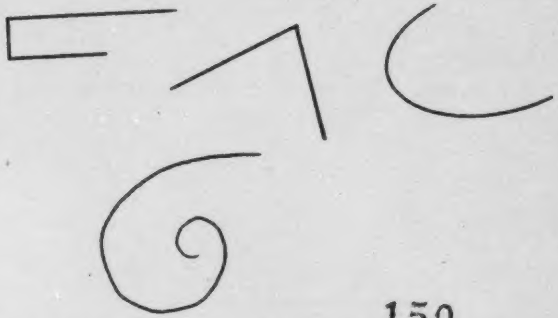


Curve

Line



Union of line segments

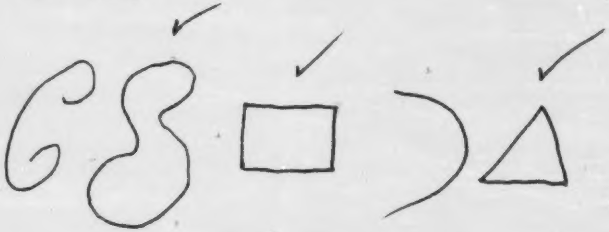
Major Topic	Behavioral Objectives	Teaching Aids
2. Closed & Open Curves (cont.)		Children learn that paths are closed curves when they can select a starting location and trace along the path to the starting location, without retracing any part of the path, except for possible intersections (where the curve crosses over itself). Some closed curves are shown in the following:  In an open curve, one must retrace at least part of the path to return to a selected starting location. Some open curves are shown in the following: 

Major Topic

2. Closed & Open Curves (cont.)

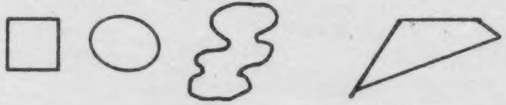
Behavioral Objectives

(c) Given a set of plan figures, the child can distinguish the simple closed figures and identify the inside and outside regions. For Example:

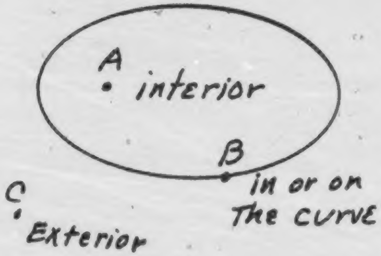


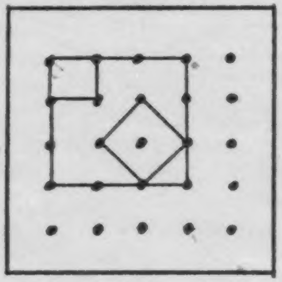
Teaching Aids

Closed curves that do not cross over themselves are called simple closed curves. Some simple closed curves are illustrated in the following:



A simple closed curve separates a plane into three sets of points--the sets of points inside the curve (the interior), the set of points in the curve (the curve itself), and the set of points outside the curve (the exterior). Points A, B and C are classified respectively as interior, in or on the curve and exterior as shown in the following:



Major Topic	Behavioral Objectives	Teaching Aids
D. CONGRUENCY		
1. Plane Figures & line segments	(a) The pupil will select congruent, plane figures as figures which fit on one another from models of a variety of figures.	Use models made of cardboard, plastic, or construction paper.
	(b) The pupil will draw or cut from pipe cleaner or wire, models of line segments which are congruent to the original model.	
E. CONSTRUCTIONS		
1. Plane Figures Circle Square Rectangle Triangle	(a) Given a pegboard and rubber bands, the child can construct a square, rectangle, triangle, and line segment.	The teacher should require a variety of examples. This may be done by asking: "How many different squares can you construct on the pegboard?"
Line Segments	(b) The child can make rough pencil and/or chalk drawings (outlines) of circles, squares, rectangles, triangles, and line segments.	
	(c) Using a straightedge, the pupil can draw a recognizable square, rectangle, and triangle.	By using the corner of a page as a model of a right angle, assistance can be given pupils for making squares, rectangles and triangles.

Major Topic	Behavioral Objectives	Teaching Aids
F. METRIC 1. Length	(a) The child can demonstrate his understanding of the concept of length by drawing a line segment of given length (whole units).	
Number line	(b) Using a ruler, the child can construct a number line and label the points with the whole numbers.	
	(c) Pupils will estimate perimeters or lengths of objects Pupils will verify by measuring the objects using metric rulers.	Perimeters may be found by wrapping or spanning string and then measuring the string.
	(d) Using a ruler, the child can construct a line segment of given length. For example: Draw a line segment 15 cm long.	
	(e) Child can select and apply a unit of measure repeatedly (such as a pencil, book edge, etc.) to determine the length of an object or compare the length of different objects.	
	(f) Students will measure lengths of objects in centimeters and meters.	Pupils will measure sides of simple polygons. Add the measure of each length to find the total perimeter.
2. Area	(a) Given a problem, such as "How large is this surface?" the pupil will adopt a unit of measure and apply it in a consistent manner. After sufficient applications, he will report the area of the surface in terms of the standard he has selected.	A child may be assisted to appreciate the greater accuracy that may result from using smaller and uniform units. Attempts should be made to elicit this conclusion from the pupil. He should not be told this conclusion at this grade level.

Major Topic	Behavioral Objectives	Teaching Aids
2. Area (cont.)		Teaching Aid for Behavioral Objective (a) (continued): He may use a lunch ticket, book cover, etc. to measure and report surface areas. He should recognize that he needs to report--(1) the unit used to determine the area, and (2) the number of times the object was applied to measure the area.
	(b) Given a unit of area on the pegboard (not necessarily a square), the child can construct a figure having a value of two, three, or more.	Ask children how many ways they can construct an area containing two. For additional information, see Geoboard references of Mathematics in Primary Schools by The Association of Teachers of Mathematics, published by Cambridge University Press.

MEASUREMENT AND ESTIMATION

GRADE TWO

Major Topic	Behavioral Objectives	Teaching Aids
A. CONCEPTS OF MEASUREMENT		
1. Process of Measuring Length Selection of (arbitrary) Counting	Given an object with the property of length, the student can select a suitable unit of length and measure the object by counting the number of units needed to match the length of the object.	Children should be encouraged to "measure" familiar objects in their classroom, using units they invent themselves. If they use their feet or hands, questions will arise naturally, "Is everyone's foot the same size?" "Are some people"s hands bigger than others?"
2. Graphs	The student can tell from picture graphs which group is larger or which is smaller.	Have a collection of different picture graphs for use in the classroom.
B. MEASUREMENT OF PHYSICAL PROPERTIES		
1. Time Clock Hour, half-hour, Quarter-hour.	(a) Using a clock, the student can tell time on the hour, half-hour, and quarter-hour.	The Clock Game Have on hand a large clock face and individual clock-faces that the children have made from paper plates. (See Addenda, page 328). Use individual cardboard clocks with moveable hands. You should have students make their own. 12 1 2 3 4 5 6 7 8 9 10 11
Calendar Day Week Month	(b) The student can use day, week, and months as units of measure of time. Example: There are ____days in a week.	The Calendar Game (See Addenda, page 328)

2. **Money**
Penny, nickel, dime, quarter, half-dollar

(a) The student can identify and name pennies, nickels, dimes, quarters, and half-dollars and state the value, in cents, of each coin.

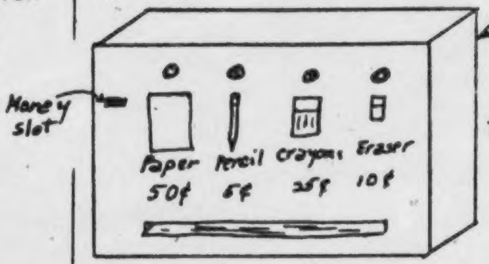
Games: How Much Money Have I?
Shopping Trip - See addenda.

How many different combinations of coins make 13¢? Make a chart.

dimes	nickels	pennies	total coins
		13	13
	2	3	5
1		3	4
	1	8	9

(b) Given a set of coins (pennies, dimes, quarters, etc.), the student can determine and name the total value of the coins in both cent notation and decimal notation. (e.g., 90¢ and $.90)

Toy money and make believe vending machines may help.



Let a student play as the machine

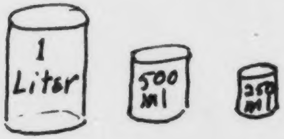
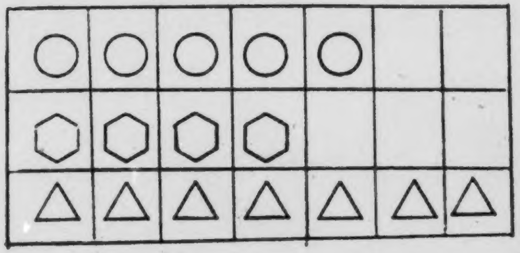
3. **Linear Measurement**
Length
Ruler
centimeter
meter

The student can measure a given line segment or object to the nearest whole unit.

Develop relative concepts of length with questions as: Which is longer, a piece of chalk or the eraser beside it? Your shoe or your foot? A crayon or a new pencil? etc.

Have pupils estimate the lengths of some objects by placing one beside the other and then determine the actual measurements with their rulers. Articles should not exceed 30 centimeters.

MEASUREMENT AND ESTIMATION GRADE TWO

Major Topic	Behavioral Objectives	Teaching Aids
4. Volume Liquid measure 1/4 liter, 1/2 liter, liter	Using a 250 ml measure, the student can measure the capacity of a given container to the nearest whole unit.  	Have 1/4 liter, 1/2 liter, liter, 4 liter containers near sink for children to do their own experiment. The students should be exposed to the comparison of 1/4 liter with 250 milliliter. Have children bring containers of different liquid measures (paint cans, cream bottles, juice cans, etc.) and have them determine how many 1/4 liters, 1/2 liters they hold.
5. Weight -- Mass Balance	(a) Given two objects and using a simple balance, the student can determine and name which is heavier and which is lighter.	Use a coat hanger balance and small objects for the students to weigh to determine which is heavier or lighter. See addenda.
Scales kilograms	(b) Using simple scales, the student can determine and name the weight of a given object to the nearest kilogram.	Bathroom scales could be used for determining weights of larger objects.
6. Temperature Thermometer Degree (Celsius)	Based on experiences with reading scales on a thermometer, the student can record temperatures inside and outside the classroom and answer questions such as: Is it cooler (colder) or warmer (hotter) today than yesterday?	Use a large homemade cardboard thermometer for demonstration purposes. Mark a Celsius scale on the cardboard and have a moveable ribbon or cord, a part of which is red, to represent the liquid in the thermometer.
7. Graphs	The student can make picture graphs appropriate to grade level, to show which quantities are larger and which are smaller.	 Use a flannel board for displaying objects.

Major Topic	Behavioral Objectives	Teaching Aids
C. REMAINING MEASURES		
1. Comparison Units dm is the abbreviation of decimeter or 1/10 of a meter	The student can express the relationships between units of measure appropriate to the grade level and can rename a measure in other units. For Example: 1 dm is (longer, shorter) than 1 cm. 1 liter = __________ 250 ml 1 half-dollar = ________ dimes	

Major Topic	Behavioral Objectives	Teaching Aids
A. THE NATURAL NUMBERS COUNTING & NUMERATION		
1. Concepts Related to Counting 1-10,000	(a) Given a number greater than 1000, the student can verbalize his notion of the quantity involved.	Prepare a display to demonstrate how large a set is that contains 10,000 objects. This can be done by typing 2000 X's on a ditto master and then duplicating the required number of pages. A convenient format would be 40 rows with 50 x's in each row. These sheets could then be mounted on a wall to impress the students with the size of 10,000.
1-10,000	(b) Given a number, the student can identify the numbers which precede and follow it.	Refer to Grade I, Section A-1-(b)
1-10,000	(c) The student can identify and name odd and even numbers.	Refer to Grade I, Section A-1-(a).
2. Counting the Number In a Set		
Collections	(a) Given a description of a set, the student can distinguish between members of the set and things which are not members.	It is desirable for students to have practice with a variety of sets such as multiples* of 2, 3, 5, 2 and 3 3 and 5 and any small subset of the prime numbers.**
One-to-one Correspondence	(b) Given two equivalent sets of objects or pictures, the student can demonstrate a one-to-one matching between members of sets by constructing lines or by physically associating the objects.	* Multiples of 2 $\{0, 2, 4, 6, 8, 10, 12...\}$ ** Prime number is a whole number greater than one that is not divisable by any other number than itself and one. $\{2, 3, 5, 7, 11, 13, 17, 19, 23, . . .\}$

Major Topic	Behavioral Objectives	Teaching Aids
One-to-many correspondence	(c) The student can describe familiar physical situations in which a one-to-many matching occurs. For example: One foot to five toes or one coat to two sleeves.	Refer to Grade II, Section A-2-(c).
Equivalent & non-equivalent sets.	(d) Given two sets (objects, pictures, verbal description), the student can identify them as equivalent or non-equivalent, and if non-equivalent, tell which set has more and which has fewer members.	Use sets of physical objects for students to match until they develop their skills to where they can solve problems involving more abstract sets such as the set of multiples of 5, less than 30 compared to the multiples of 3, less than 20.
Empty Set	(e) The student can describe verbally a set which has no members, such as the set of all native Martians in the classroom.	Encourage students to suggest mathematical examples of the empty set such as the set of all two digit integers less than ten, or the set of triangles that are also squares.
Subsets	(f) Given a set, such as all students in the classroom, the student can identify a subset, such as the students wearing tennis shoes.	Encourage students to suggest mathematical examples of subsets such as the set of squares is a subset of the set of rectangles or the set of multiples of six is a subset of the set of multiples of three.
Cardinal Numbers 0-10,000	(g) The student can determine the cardinal number of a given set.	Encourage students to develop shortcuts for counting the members of a set. For example, the number of squares on a checkerboard, the number of windowpanes in a room, the number of desks in a room, etc.

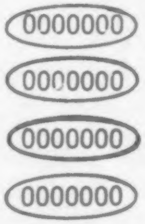
Major Topic	Behavioral Objectives	Teaching Aids
3. Reading Recognizing, & Writing Numbers		Encourage students to find as many different names as they can for a given number.
Names for Numbers	(a) The student can identify, name, read, and write many different names for the same number. For example: 14 = 10 + 4 = XIV = 2 x 7 = 20 - 6, etc.	From a list, have the students select the different names for a given number.
Whole Numbers 0-10,000	(b) The student can identify, name, read, and write numerals for whole numbers.	
	(c) The student can read and write number words through nine hundred ninety-nine.	
4. Using Ordinal Names		
Order relations	(a) Given a set of whole numbers, the student can write them in order from least to greatest and from the greatest to the least.	
	(b) The student can write the symbols <, >, ≠, =, to express the relationship between two given numbers.	Combine practice with basic facts and order relations by using problems such as: 3 + 4 ____ 5 + 3 or 9 - 2 ____ 4 + 4.
Ordinal Numbers beyond twentieth	(c) Given a sequence of objects, events, etc., the student can identify the position in space or time of a particular object or event by saying: "This (object) is fourth" or "This (event) happened second."	Given a rectangular array, have the students locate the 37th element, etc. Have the students determine the date of a given day of the year such as the 50th day of the year, etc.

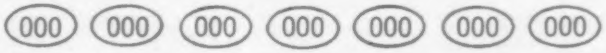
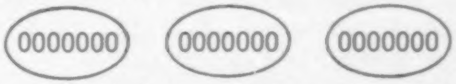
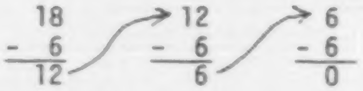
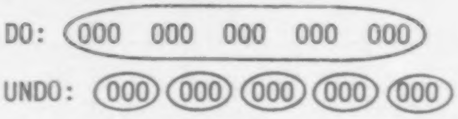
Major Topic	Behavioral Objectives	Teaching Aids
5. Place Value		
4-digit numerals	(a) Given a numeral such as 8073, the student can identify, name, and distinguish the numerals that are in the ones, tens, hundreds, and thousands places.	The abacus with nine discs on each loop can be very useful. One variation in the use of the abacus is to have the students listen for the disc to be flipped over rather than watching.
	(b) Given a numeral, the student can name the place value for each digit.	Refer to grade Two, Section A-5-(a). (Use four sets of numeral cards for four digit numbers)
Expanded notation	(c) Given a numeral such as 4444, the students can write the expanded numeral (4000 + 400 + 40 + 4).	
6. Non-Decimal Numeration		
Roman numerals through XV	(a) Given a numeral such as 12, the student can write the Roman numeral XII.	Prepare a chart with the Arabic numeral equivalent to a Roman numeral.
	(b) Given a Roman numeral such as XIV, the student can write the Arabic numeral 14.	
7. Rounding & Estimation		
	(a) Given any 2-digit numeral, the student can round to the nearest tens.	Use tongue depressors, both singly and bundles of ten. The number line is also very useful in developing this concept.
	(b) Given two 2-digit numerals, the student can estimate the sum to the nearest tens.	Flash cards could be useful in developing students' ability to rapidly estimate the sum of two numbers.

Major Topic	Behavioral Objectives	Teaching Aids
B. THE NATURAL NUMBERS OPERATIONS		
1. Addition & Subtraction		
Definition	(a) Given an addition or subtraction problem such as 5 + 7 = ____ or 14 - 8 = ____, the student can design a simple experiment involving sets or a number line to determine and name the sum or difference of the numbers.	Have a variety of materials available for the students to count if they encounter difficulty with the basic facts.
Inverse relationship	(b) Given an addition equation such as 13 + 24 = 37, the student can write the two related subtraction equations, 37 - 24 = 13 and 37 - 13 = 24.	Use two digit numeral cards and operation cards to physically demonstrate the inverse operations. Use the cards until students are thoroughly familiar with this concept.
	(c) Given an addition equation with a missing addend such as ____ + 16 = 157, the student can write the related subtraction equation, 157 - 61 = ____.	The cards described above could be useful in developing this objective.
	(d) The student can check subtraction problems by addition.	
Basic facts through sums of 18	(e) Given any single-digit addition or subtraction combination, the student can immediately* name the sum or difference.	Use as many different activities as necessary to develop speed of response by students. These could include timed quizzes, flach cards, tachistoscopes, number fact records, tape recorded problems, etc.

* Immediately is defined as 5 seconds or less.

Major Topic	Behavioral Objectives	Teaching Aids
Algorithms Column addition & subtraction (4-digit numerals) without regrouping	(f) The student can name the sums, differences, and missing digits for problems such as: $\begin{array}{r} 2405 \\ 71 \\ 301 \\ \underline{+4012} \end{array}$ $\begin{array}{r} 5673 \\ \underline{-4052} \end{array}$ $\begin{array}{r} 7\ \square\ 6 \\ \underline{-\square\ 3\ 4} \\ 2\ 6\ \square \end{array}$	It is felt that practice in column addition and subtraction is necessary. However, care should be exercised in avoiding an excessive number of problems for only providing drill. This drill can be provided in more interesting activities such as cross number puzzles, puzzle type problems, etc. Refer to Grade II-B-(d), and modify to suit the operation and the size of your numbers.
Column addition & subtraction (3-digit numerals) with regrouping.	(g) The student can name the sums and differences for problems such as: $\begin{array}{r} 453 \\ \underline{+\ 439} \end{array}$ $\begin{array}{r} 694 \\ \underline{+\ 19} \end{array}$ $\begin{array}{r} 570 \\ \underline{-\ 147} \end{array}$ $\begin{array}{r} 803 \\ \underline{-\ 227} \end{array}$	Subtraction can be done as follows. Be sure to try these out yourself first. You may leave out all but four numbers and still fill in the problem. (try it) ⊖ 25 13 12 11 7 4 14 6 8
Other notation	(h) The student can identify and name sums, differences, missing addends, and missing operational signs in problems written in both horizontal and vertical notation.	
2. Properties of Natural Numbers Properties Commutative property of addition.	(a) The student can solve equations such as: 17 + ____ = 32 + 17 or ____ + 16 = 16 + ____	Use the numeral and operation cards to demonstrate the physical operation of interchanging the position of these two numerals.

Major Topic	Behavioral Objectives	Teaching Aids
Associative property of addition	(b) Given a problem such as 8 + 49 + 1 = ____, the student can indicate the grouping of addends which will make the addition easiest by enclosing the 49 and 1 in parentheses. For example: 8 + (49 + 1) = 58.	Have students place the appropriate number of felt pads on a flannel board marked in columns of ten. The student can then regroup the pads so as to make the easiest way of finding the sum.
	(c) Given a problem such as 5 + 7 + 2 + 3 = ____, the student can demonstrate how to find the sum in the easiest way by rearranging the addends.	
Identity element for addition	(d) The student can solve equations such as: 41 + ____ = 41; 37 + ____ = 37 - ____; 17 - ____ = 0; and 53 - ____ = 53.	For students having trouble with the identity element, have them work with blocks or Cuisenaire rods the types of problems illustrated by the suggested examples.
3. Multiplication & Division		
Definition of Multiplication	(a) Given a multiplication problem such as 4 x 7 = ____, the student can design a simple experiment involving the union of four equivalent disjoint sets each with seven members to determine and name the product of the numbers.	For example:  4 x 7 = 28 See addenda on multiplication.
	(b) The student can solve equations such as 3 x 7 = 7 + ____ + ____ and 8 + 8 + 8 + 8 + 8 = _____ x 8 and use sets or skip count on the number line to prove his results.	Use tongue depressors, tally marks, array of dots, etc. as necessary to illustrate this concept.

Major Topic		Behavioral Objectives	Teaching Aids
Definition of Division	(c)	Given a set of 21 objects, the student can partition the set into equivalent disjoint subsets each having three members and name how many such subsets can be formed. 	A variety of objects such as chops, sticks and bottle caps should be available for students to manipulate as necessary. Have the students write the mathematical statement illustrated by the experiment. 21 - 3 = 7
	(d)	Given a set of 21 objects, the student can partition the set into three equivalent disjoint subsets and name how many members are in each subset. 	The teacher should carefully note the difference between the types of division problems illustrated in (c) and (d). The student should have experience with counters in solving both types of problems.
	(e)	Given a division problem such as 36 - 9 = ____, the student can design a simple experiment involving set partition (see objectives (c) and (d) above) to determine and name the quotient.	
	(f)	The student can repeatedly subtract 6 from 18 until there is a remainder of 0, name how many times the 6 can be subtracted, and write the related division equation.	For example:  6 can be subtracted from 18 three times. 18 - 6 = 3
Inverse relationship	(g)	The student can demonstrate by using sets or a number line that dividing 15 by 3 "undoes" multiplying 5 by 3.	For example:  5 x 3 = 15 15 - 3 = 5 Use manipulative materials as necessary to illustrate this concept.

NUMBER AND OPERATIONS GRADE THREE

Major Topic	Behavioral Objectives	Teaching Aids
Basic facts Through products of 45 (1-digit factors)	(h) Given a multiplication or division combination such as 4 x 8 or 24 ÷ 6, the student can immediately* name the product or quotient and use sets or a number line to prove his result.	
Through products of 81 (1-digit factors)	(i) Given a multiplication or division combination such as 7 x 7 or 72 ÷ 9, the student can use sets or a number line to determine and name the product or quotient.	Refer to Grade two, Section B-3-(e)
	(j) The student can identify and name products, quotients, missing factors, and missing operational signs in problems written in both horizontal and vertical notation.	Use numeral cards with operational cards so that students can physically move cards to illustrate both horizontal and vertical notation. For example: $\boxed{8}\boxed{x}\boxed{7}\boxed{=}\boxed{5}\boxed{6}$ $\begin{array}{cc} & \boxed{8} \\ x & \boxed{7} \\ \hline \boxed{5} & \boxed{6} \end{array}$
Algorithms Factors of 10, 100, & multiples of 10 & 100	(k) The student can name the products and quotients for problems such as: 7 x 1 = ____ 7 x 4 = ____ 7 x 10 = ____ 7 x 40 = ____ 7 x 100 = ____ 7 x 400 = ____ 3600 ÷ 1 = ____ 3600 ÷ 9 = ____ 3600 ÷ 10 = ____ 3600 ÷ 90 = ____ 3600 ÷ 100 = ____ 3600 ÷ 900 = ____	The teacher should stress that this type of problem is an application of the associative principle. For example: 2 x 30 = 2 x (3 x 10) = (2 x 3) x 10 = 6 x 10 = 60. 7 x 40 = 7 x (4 x 10) = (7 x 4) x 10 = 28 x 10 = 280 The student should have considerable practice with this concept.

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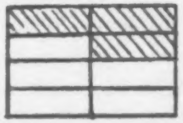
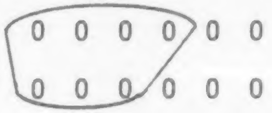
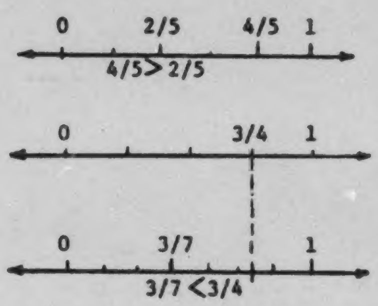
* Immediately is defined to be 5 seconds or less

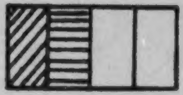
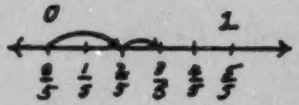
Major Topic	Behavioral Objectives	Teaching Aids
Multiplication Vertical notation (3-digit factor by 1-digit factor) with & without regrouping	(l) The student can name the products for problems such as: $\begin{array}{r}132\\ \times 3\\ \hline\end{array}$ $\begin{array}{r}217\\ \times 4\\ \hline\end{array}$ $\begin{array}{r}132\\ \times 7\\ \hline\end{array}$ $\begin{array}{r}704\\ \times 6\\ \hline\end{array}$	Note: Any of the following algorithms may be used. $\begin{array}{rrrrr}100 & + & 30 & + & 2\\ & \times & & & 7\\ \hline 700 & + & 210 & + & 14\end{array} = 924$ $\begin{array}{r}132\\ \times 7\\ \hline 14\\ 210\\ \underline{700}\\ 924\end{array}$ $\begin{array}{r}132\\ \times 7\\ \hline 924\end{array}$ $7 \times (132) = 7 \times (100 + 30 + 2)$ $= 700 + 210 + 14 = 924$
Long Division (3-digit dividend by 1-digit divisor) without remainder.	(m) The student can name the quotients for problems such as: $8\overline{)240}$ $9\overline{)666}$ $7\overline{)308}$ $5\overline{)625}$	Note: Any of the following algorithms may be used: $\begin{array}{rr}5\overline{)625} & \\ \underline{500} & 100\\ 125 & \\ \underline{100} & 20\\ 25 & \\ \underline{25} & 5\end{array}$ $\begin{array}{r}\left.\begin{array}{r}5\\ 20\\ 100\end{array}\right\}125\\ 5\overline{)625}\\ \underline{500}\\ 125\\ \underline{100}\\ 25\\ \underline{25}\\ 0\end{array}$ $\begin{array}{r}125\\ 5\overline{)625}\\ \underline{500}\\ 125\\ \underline{100}\\ 25\\ \underline{25}\\ 0\end{array}$ Students should have an opportunity to manipulate counters to illustrate a problem. It should be stressed that the algorithm is a mathematical description of the activity performed with the counters.

Major Topic	Behavioral Objectives	Teaching Aids
4. Properties Commutative property of multiplication	(a) The student can demonstrate by using sets or a number line that 6 x 4 = 24 and 4 x 6 = 24	Refer to Grade II, Section B-4-(a)
	(b) The student can solve equations such as: 7 x 2 = ____ x 7 and ____ x 8 = 8 x ____.	Refer to Grade II, Section B-4-(a)
Associative property of multiplication	(c) The student can solve equations such as: (2 x 3) x 4 = ____ x 4 = ☐ (5 x 2) x 3 = ____ x (2 x 3); and 8 x (1 x 3) = ____.	Give the students numerous problems of this type to stress that the manner in which the factors are grouped does not affect the product. For example: (2 x 3) x 4 = 2 x (3 x 4)
Identity element for multiplication	(d) The student can solve equations such as: 1 x 18 = ____; 4 x ____ = 4; 12 - 12 = ____; 14 - 1 = ____; and 3 x ____ = 3 - ____.	The properties of the identity element may be made clear to the student by the use of frame arithmetic, e.g. 1 x ☐ = ☐ ; ☐ ÷ ☐ = 1; (☐ ≠ 0; ☐ ÷ 1 = ☐ .
Multiplicative property of 0	(e) The student can solve equations such as: 15 x 0 = ____; ____ x 52 = 0; and 13 x 0 x 7 = ____.	Stress that 3 x 0 can be interpreted as 0 + 0 + 0 = 0. Also, some typical problems as follows can be helpful: If a person is paid $6 per day and works 0 days, how much money does he earn? Thus: 0 x 6 = 0 . See addenda for discussion on zero.

Major Topic	Behavioral Objectives	Teaching Aids
Distributive property of multiplication over addition	(f) The student can demonstrate by using sets that: $3 \times (2 + 4) = (3 \times 2) + (3 \times 4) = 6 + 12 = 18$	Use the counters to demonstrate this concept by actually moving them to represent the conditions of the problem. e.g. $\left.\begin{array}{l} 00\ 0000 \\ 00\ 0000 \\ 00\ 0000 \end{array}\right\} \longrightarrow \begin{array}{l} 000000 \\ 000000 \\ 000000 \end{array}$ $(3 \times 2) + (3 \times 4) = 3 \times (2 + 4) = 3 \times 6$ The use of a descriptive term such as the "do-it-to-both" rule, e.g. the 2 and 4 are both multiplied by 3.
	(g) Given a multiplication combination such as 7 x 6, the student can use the distributive property of multiplication over addition to determine and name the product.	7 sets of 6 may be thought of as 3 sets of 6 and 4 sets of 6. $7 \times 6 = (3 + 4) \times 6 = (3 \times 6) + (4 \times 6) =$ $18 + 24 = 42$ $(3 \times 6)\ \begin{array}{cccccc} 0&0&0&0&0&0 \\ 0&0&0&0&0&0 \\ 0&0&0&0&0&0 \end{array}$ $(4 \times 6)\ \begin{array}{cccccc} 0&0&0&0&0&0 \\ 0&0&0&0&0&0 \\ 0&0&0&0&0&0 \\ 0&0&0&0&0&0 \end{array}$

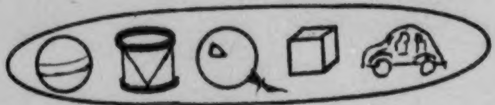
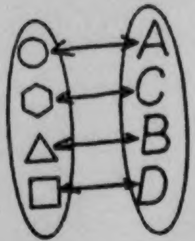
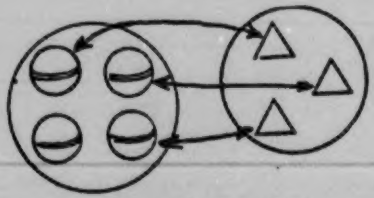
Major Topic	Behavioral Objectives	Teaching Aids
C. THE RATIONAL NUMBERS 1. Reading & Writing the Fractional Numeral		
Rational Numbers Denominators of 2-12	(a) Given a model of three-eighths, the student can identify, name, read, and write a numeral (fraction) for the rational number associated with the model.	 3/8 Students will need many different physical models of fractions such as parts of circles squares, rectangles, dozens, yards, feet, dollars, etc.
	(b) The student can construct and identify models for rational numbers.	This is the reverse of (a).
	(c) Given a set of 12 objects, the student can construct a model for 7/12 by grouping the members of the set into subsets.	For example: 

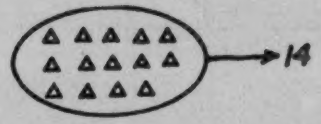
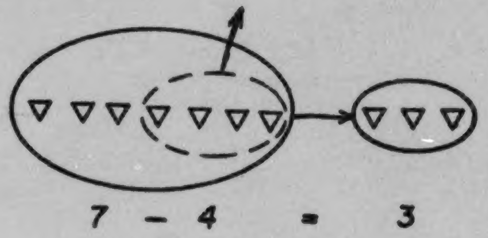
Major Topic	Behavioral Objectives	Teaching Aids
2. Equivalent Fractions	The student can determine whether two fractions are equivalent.	Use parts of circles divided into halves, thirds, fourths, sixths, and eighths to demonstrate various equivalent relations such as 2/4 = 1/2, 1/3 - 2/6, etc.
3. Ordering Fractions	Given models of two rational numbers named with fractions having the same numerators *or* the same denominators, the student can write the symbols <, >, =, ≠ to express the relationship between them.	For example:  Give the students a variety of experiences working with physical models of fractions with a common denominator. Egg cartons, packages of gum, bundles of tongue depressors and fractional parts of a dollar are very useful for demonstrating this concept.

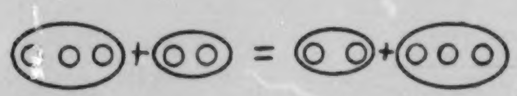
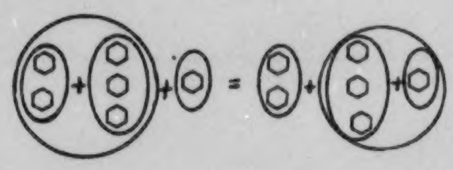
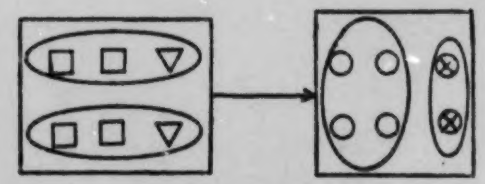
Major Topic	Behavioral Objectives	Teaching Aids
D. THE RATIONAL NUMBERS OPERATIONS 1. Addition & Subtraction of Fractions Addition		 
Definition of addition (fractions with the same denominators)	Given a model (region or number line), for the addition of two rational numbers, the student can determine and name the sum.	1/4 + 1/4 = _____ 2/5 + 1/5 = _____ Packages of gum, bundles of tongue depressors, egg cartons, fractional parts of money rectangular regions and the number line are essential to develop an understanding or this concept.

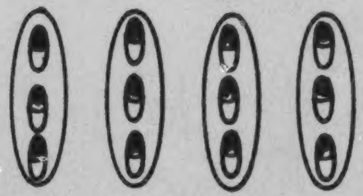
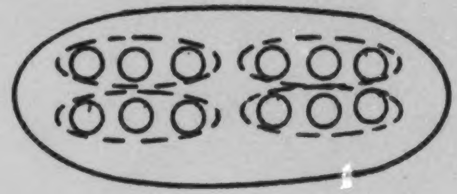
SETS, FUNCTIONS, RELATIONS, SYSTEMS, AND LOGIC

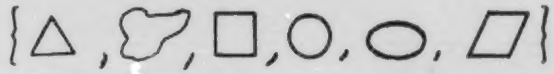
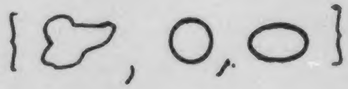
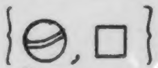
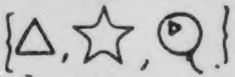
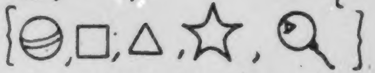
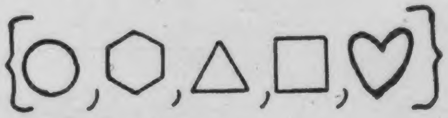
GRADE THREE

Major Topic		Behavioral Objectives	Teaching Aids
A. SETS			
1. Collections		Given a description of a set, the student can distinguish between members of the set and things which are not members.	Use flannel board to display sets. Given a set of toys, such as the following, the student can select the toys that begin with the letter b. 
2. One-to-one Correspondence		Given two equivalent sets (objects or pictures), the student can demonstrate a one-to-one matching between members of the sets by constructing lines or by physically associating the objects.	
3. One-to-many correspondence		The student can describe familiar physical situations in which a one-to-many matching occurs. For example: one foot to five toes or one coat to two sleeves.	
4. Equivalent & non-equivalent sets	(a)	Given two sets (objects, pictures, verbal description), the student can identify them as equivalent or non-equivalent, and if non-equivalent, tell which set has more and which set has fewer elements.	

Major Topic	Behavioral Objectives	Teaching Aids
Empty Set	(b) The student can describe verbally a set which has no members, such as the set of all people ten feet tall in the classroom.	To develop the concept of an empty set, have paper bags filled with different numbers of objects. Let the student reach in the bag and draw out an object. An empty set is like a bag without any objects in it.
Subsets	(c) Given a set, such as all the students in the classroom, the student can identify a subset, such as the students wearing tennis shoes.	Have a collection of pictures containing several objects of different kinds.
5. Cardinal Numbers 0-10,000	The student can determine the cardinal number of a given set.	Use a collection of objects for the student to count. 
6. Set Union	(a) Given two sets, the student should be able to join the two sets and get a third set.	A = [□,○,△] B = {♡,☆} A ∪ B = {□,○,△,♡,☆}
	(b) Given a set of elements, the student should be able to remove a set and get a remaining set and associate with this a number sentence that describes the above action.	

Major Topic	Behavioral Objectives	Teaching Aids
7. Commutative Idea	(a) The student shall be able to demonstrate if you join a set, say, of three elements to a set containing two elements, it is the same as joining a set of two elements with a set of three elements.	Use a flannel board with Cuisenaire Rods and a collection of objects. 
Associative Property	(b) The student shall be able to demonstrate the associative property through the use of sets. For example: $(2 + 3) + 1 = 2 + (3 + 1)$	Use a collection of objects. 
Distributive Property	(c) The student shall be able to demonstrate the distributive property through the use of sets. For example: The number named by (2 x 3) is the same as the number named by (2 x 2) + (2 x 1).	Use collection of objects, bottle caps, popsicle sticks, and crayons.  $2 \times 3 = (2 \times 2) + 2 \times 1 = 6$

Major Topic	Behavioral Objectives	Teaching Aids
8. Concept of Multiplication	(a) The student can describe set union as multiplication as in the example: Four three's or 4 X 3 = ☐	Use collection of objects, bottle caps, popsicle sticks and crayons. 
Set Partitioning	(b) The student shall be able to describe set partitioning as division. For example: 12 ÷ 3 = ☐	Use peg board, geo-board, collection of small objects, popsicle sticks, and egg cartons with bottle caps. Treat division as repeated subtraction; i.e., subtract 3 four times from 12 to get zero or no remainder 12 ÷ 3 = 4. Division can also be treated as the diagram indicates; i.e., 12 items can be grouped in four groups of three in each group 12 ÷ 3 = 4. 

Major Topic	Behavioral Objectives	Teaching Aids
9. Set Symbolism	(a) Given a description of Set A and a larger set (Set B) containing Set A, the student can select Set A from the larger Set. For example: Set A is the set of curved figures; And the Larger set; SET B = 	SET A = 
A()B	(b) If given two sets, the student can represent the union of the two sets in set notation. A =  B =  AUB = 	
n	(c) The student shall be able to associate a set with a numeral that names the number of elements in the set either by using set notation or simply naming the number of elements of the set.	SET C =  N(C) = 5

Major Topic	Behavioral Objectives	Teaching Aids

B. RELATIONS & FUNCTIONS

1. "What's My Rule?"

Given the game situation described in the addenda, the student will be able to determine the number that matches the question mark.

In	Out
3	7
4	8
5	9
7	?

In	Out
2	4
3	6
4	8
8	?

In	Out
(3,4)	7
(3,5)	8
(4,5)	9
(2,6)	?

See addenda.

In	Out
3	6
4	8
5	10
6	-

2. Number Patterns

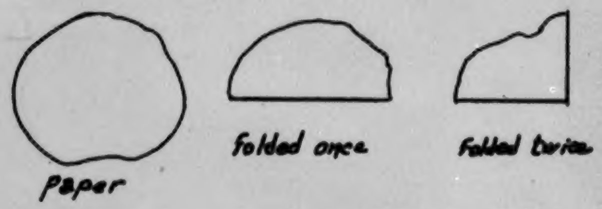
The student shall be able to complete the next two elements of the following sets.

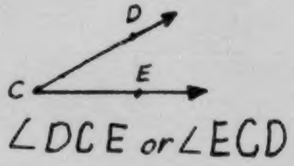
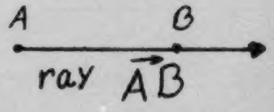
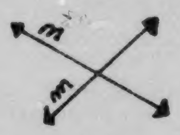
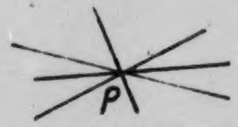
A = {1, 4, 7, 10, ___, ___, . . .}

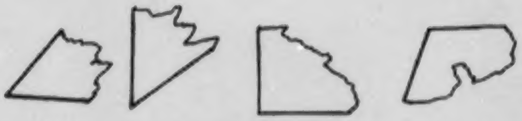
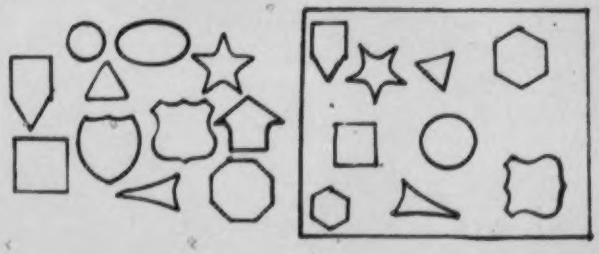
B = {3, 6, 9, ___, ___, . . .}

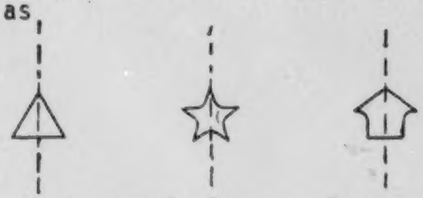
See addenda.

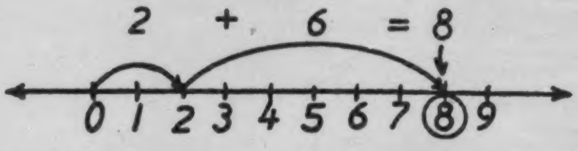
Major Topic	Behavioral Objectives	Teaching Aids
C. MATHEMATICAL SENTENCES		
1. Open, true & false sentences.	(a) The student shall be able to classify the following types of sentences as open, true or false, and answer or correct the open and false sentences. $2 \times 6 = 12; \quad 12 \div 3 = 5;$ $5 - \square = 2; \quad 2 + 5 = 7$	True, $4 \times 4 = 16$; False, $12 \div 4 = 4$; Open, $5 - \square = 2$
≠	(b) The student shall be able to use the "is not equal to" sign.	$5 - 2 \neq 1$
( )	(c) The student shall be able to use parentheses to show order in mathematical sentences involving addition, subtraction, and multiplication.	$24 - (2 \times 4) = 16$
	(d) The student can find the solution set to mathematical sentences with two variables. For example: $6 \times 3 = (\square \times 3) + (\triangle \times 3)$	Truth set or solution set = T = all the pairs of $(\square, \triangle)$ or $(\square, \square)$ that make the sentence true. $\square + \triangle = 8$; $T = \{(0,8), (1,7), (2,6), \ldots, (7,1), (8,0)\}$ $\square + \square = 8$; $T = \{(4,4)\}$

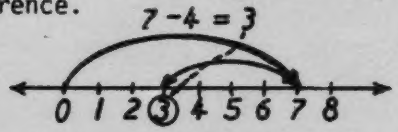
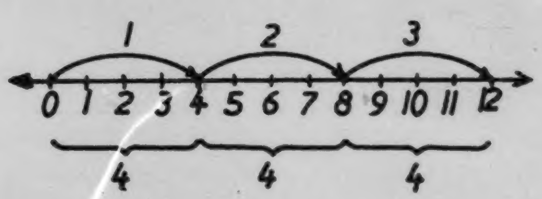
Major Topic	Behavioral Objectives	Teaching Aids
A. RECOGNITION & DESCRIPTION OF PLANE FIGURES		
1. Plane Figures Point Line Line Segment Ray Path (curved line) Angle right angle Quadrilateral square rectangle Triangle Circle center radius diameter chord	Given models of the plane geometric figures named on the left (wire, paper or flannel cut-outs), the pupil can identify, name, and distinguish among these figures.	The pupil may be helped to compare rays to light from a searchlight, an arrow in continuing unchanging flight, etc. Rays are half lines which begin at a point and extend in one direction. A right angle is formed by perpendicular lines or line segments which share a common intersection or vertex, and is similar to a square corner. An angle is formed by two rays with a common endpoint. A right angle may be formed by folding a piece of paper then folding it once again so that the creases align.  To construct a right angle model, child can fold the paper as illustrated. He will also recognize corners of a page as a model of a right angle.

Major Topic	Behavioral Objectives	Teaching Aids
B. RECOGNITION & DESCRIPTION OF SOLID FIGURES		
1. Space Figures Cube Sphere Cylinder Cone	Given models of cubes, spheres, cylinders, and cones (wood or plastic solids, rolled paper, etc.), the child can identify, name, and distinguish among these space figures.	Use blocks, plastic models, tin cans, balls, and other objects.
C. SETS OF POINTS		
1. Location	Given a verbal description of a precise location in the classroom, the child can locate and identify the point. For example: Where is the place the floor and these two walls meet?	
2. Plane Figures	(a) The pupil can draw, name, and describe rays and angles.	 
	(b) The pupil can describe and explain that there is only one line through two points and that two lines can intersect at only one point.	
	(c) The pupil can illustrate that many lines may pass through one point.	 many lines through one point

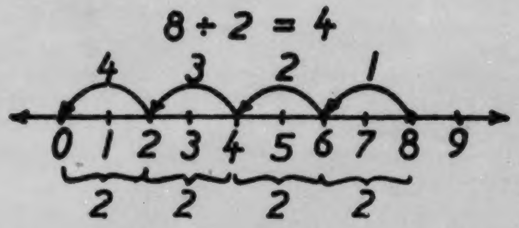
Major Topic	Behavioral Objectives	Teaching Aids
D. CLOSED & OPEN CURVES & REGIONS		
1. Congruency	(a) The pupil can match models of angles to determine which are congruent.	 The teacher will recognize that the angles formed by two rays but we compare to check congruency by using cutouts. These cutouts technically include portions of the interior of the angles. Children may use a tracing to place one angle upon another angle. In this way, they may tell if the measure of one angle is greater than, less than, or equal to the measure of another angle.
	(b) The pupil can demonstrate that a rectangular sheet of paper can be divided in two or more congruent parts through folding.	
	(c) The pupil can match a cutout figure to the space from which it was cut.	Construction paper with geometric figure shapes removed may be fastened to the bulletin board. The student is to match the pieces with the spaces from which they came. 

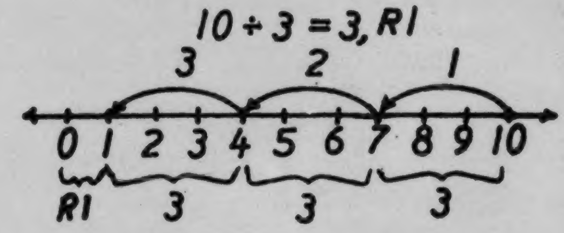
Major Topic	Behavioral Objectives	Teaching Aids
Congruency cont.		Both additional unmatched pieces and cutout hollows may be included which do not have mates. Technically, the cutout piece is congruent to the hollow; that is, only for those pieces that have mates.
2. Symmetry	The pupil can choose these figures which possess symmetry with respect to a line by folding paper containing symmetrical figures such as  along their vertical axes of symmetry.	Symmetry is evidenced in the human body (arm for arm, ear for ear, etc.) and in certain views of some houses, animals, trees and flowers. Arrangement of Cuisenaire rods or colored blocks or tiles may possess symmetry as well as figures constructed on the geoboard. Use of art materials, creating half a design or picture (as a butterfly) folding and impressing the design from folded paper, then unfolding to reveal symmetry. Snowflakes are an illustration of symmetry. By using a mirror, children can test a design for the presence of symmetry and can note how many axes of symmetry there are. Capital letters may be cut from construction paper by using lines of symmetry for letters having them. Some letters possess two or more lines of symmetry.

Major Topic	Behavioral Objectives	Teaching Aids
3. Similarity	The pupil when asked to name geometric figures that possess similarity will name points, line segments, squares, and circles.	All congruent figures are similar but not all similar figures are congruent. Similar figures have the same shape. Congruent figures have the same size and the same shape.
4. Construction--Approximate & Exact	The student can demonstrate his understanding of the concept of length by drawing a line segment of given length.	
5. Coordinate Systems & Graphs	(a) Given a line and a point on that line, the pupil can mark it with equally spaced slashes and make a one-to-one correspondence between the slashes and the whole numbers called coordinates of the points.	The whole number set is $\{0, 1, 2, 3, \ldots\}$
	(b) Given a number line, the pupil can use the line to illustrate simple addition and subtraction problems.	To add on the number line, the pupil will start at zero and move (make a jump) to the right on the number line the number indicated on the first addend and from that point make a second jump to the right the number indicated by the second addend. The number associated with the landing point is the sum. 

Major Topic	Behavioral Objectives	Teaching Aids
5. Coordinate Systems & Graphs		To subtract on the number line, the pupil will start at zero and jump to the right the number indicated by the minuend. Starting at that point, the pupil will make a jump to the left (subtract) indicated by the subtrahend. The number associated with the landing point is difference. 
	(c) The pupil can use the number line to illustrate multiplication problems.	See Grade Two, Section B-4 (b) and Grade Five D-3 (a). To multiply on the number line, the pupil will start at zero and make equal jumps to the right (repeated addition) the length indicated by the number being multiplied and the number of jumps being indicated by the multiplier. For example: 3 x 4 = 12 

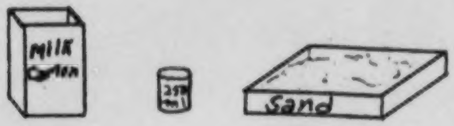
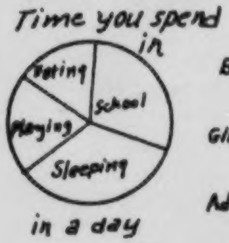
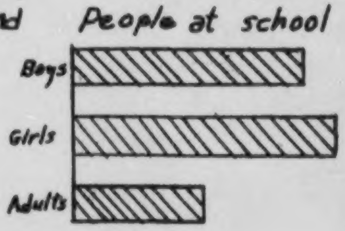
Major Topic	Behavioral Objectives	Teaching Aids
5. (Continued)		To divide on the number line, the pupil will start at the number on the number line indicated by the dividend. He will make jumps to the left (repeated subtraction) approaching zero. Each jump is the length of the divisor. He will continue to move in jumps until he either lands on zero or the length between the last landing place and zero is less than the divisor. The number of jumps is the quotient and any remaining less than the divisor is the remainder.





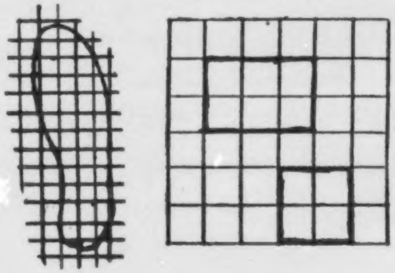
Major Topic	Behavioral Objectives	Teaching Aids
E. METRIC		
1. Length	Pupil will measure distances using paces, trundle wheel, and appropriate units to convey distances to others.	A trundle wheel may be made from pizza discs or similar wood circles marked so that revolutions can be counted. If made of light wood and in one meter circumference, pupils may be asked to measure lengths of playgrounds, distances walked from house to school. Use of meters in kilometers can be made.
2. Area	Given a rectangular surface to measure, the pupil will adopt a standard unit and measure the surface.	Pupil can experiment with different units as squares, rectangles or triangles to report surface measures. For example: A desk top may be measured by application of the textbook to the surface area of the desk. A desk can be covered by 10 books; the area of the desk would be 10 books. In first efforts, if there is a need to determine a logical measure for a region, the teacher can give children a dittoed sheet with various closed curves of varying size and shape. The child can be supplied with a set of squares cut out from square paper. (For convenience in handling, some of the squares need not be cut apart.) The child then places squares on the dittoed sheet to see how many squares cover the region. If the teacher believes that the need for actually handling of the squares by the child is not so important, then the figures might be drawn on.

Major Topic	Behavioral Objectives	Teaching Aids
2. Area (cont.)		or dittoed onto . . . a squared sheet so that now the child has only to count the squares lying inside the closed region to develop the idea that it is easier to determine a more precise measure if smaller square units are used. The teacher might use colored acetate squares for use with the overhead projector. After irregular regions are used, then use a rectangle which covers exactly a certain number of squares. We want the child to see intuitively that most regions have no formula for area. In most cases, to find an area, we must count squares and approximate fractions of squares. Then, in a few cases, such as for a rectangular or triangular region, we see that we don't have to count . . . . directly. We can determine the total number of rows of squares in the rectangle by the number of squares in each row.

Major Topic	Behavioral Objectives	Teaching Aids
A. CONCEPT OF MEASUREMENT		
1. Process of Measuring Length, Area, Volume Selection of unit (arbitrary) Counting	(a) Given an object or picture with the property of length, area, and/or volume, the student can select a suitable unit and count to measure the property of the object.	Children should be encouraged to "measure" familiar objects in their classroom, using units they invent themselves. They should measure the volume of boxes or other containers.  How many 250 ml containers of sand does it take to fill the milk carton?
2. Concept of graphs	(a) The student can identify which quantities are larger or smaller by looking at bar, circle, and picture graphs.	Have a collection of different graphs, let the student examine the graphs and find the largest and smallest quantities represented on the graph.  

Major Topic	Behavioral Objectives	Teaching Aids
B. MEASUREMENT OF PHYSICAL PROPERTIES		
1. Time Clock Hour, half-hour, quarter-hours, minute	(a) Using a clock, the student can tell time to the nearest minute.	Play the Clock Game but expand it to include minutes. See addenda. Use demonstration clock. Headline stories that involve time.
Calendar Day, week, month	(b) The student can give the correct solution to questions such as: What day is 20 days from May 17?	A Calendar Game. See addenda.
2. Money United States coins United States bills	(a) The student can make change for any purchase up to $1.00.	Shopping Trip. See addenda. Pick a lunch. Choosing Blindly.
	(b) The student can identify, name and state the value of all United States coins and bills.	
	(c) Given pictures of coins and bills of different denominations, the student can determine and name the total value in decimal notation.	The student can make a chart showing different combinations of coins that will make a dollar.
3. Linear Measurement Length Ruler, centimeter meter	The student can measure a given line segment or object to the nearest half-unit.	Have the students measure their book, desk, pencil, length and width of classroom blackboard. They should measure the smaller objects to the nearest centimeter.

Pennies	Nickels	dimes	quarters	half dollars	Total coins
100					100
95	1				96
90	2				92
90		1			91
etc.					

Major Topic		Behavioral Objectives	Teaching Aids
4. Area Square Units		The student can determine and name the area of a given rectangular region by counting the number of area units (square regions) needed to cover the region.	Use Geo-board in the discussion of area. Checkboard grid with clear plastic cutouts of squares and rectangles. Trace a foot on large grid paper and count the squares and partial squares. 
5. Volume Cubic units	(a)	The student can determine and name the volume of a given right rectangular space figure by counting the number of volume units (cubes) needed to fill the space.	Get a set of paper or wooden cubes that the students could use to count the number of them that it would take to fill a box. (Sugar cubes, Cuisenaire rods, multibase arithmetic blocks can be used.)
Liquid measure 250 ml, 500 ml, 1 liter	(b)	The student can measure the capacity of a given container to the nearest whole unit.	Have 250 milliliter, 500 milliliter and liter containers for students to use in measuring the volume of other containers such as paint cans, milk cartons, juice cans.
6. Weight-Mass Scales grams, kilograms		Using simple scales, the student can determine and name the weight of a given object to the nearest 10 grams.	A collection of various objects (rocks, stones, blocks, marbles, etc.) to a discussion of comparative weights and types of weighing instruments. Have the children lift object and guess its weight, and place the objects in rows according to relative guesses. Then weigh each object.

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Major Topic	Behavioral Objectives	Teaching Aids
7. Temperature Thermometer Degree (Celsius)	The student can record temperature inside and outside the classroom and identify and name the temperature at which water freezes (freezing point).	Use cardboard demonstration model. See addenda.
8. Graph	The student can make simple picture and bar graphs to show comparison of quantities. The student can locate a point on the road map.	Road maps for each child. Chart - Scores of Games

	John	Bill	Dan	Ed	Carl
Game 6	31	71	54	34	36
Game 5	63	18	64	29	81
Game 4	50	76	56	67	58
Game 3	39	43	47	34	52
Game 2	34	26	27	54	23
Game 1	28	27	81	60	47

Locate Dan's score for the third game.

C. RENAMING MEASURES

Major Topic	Behavioral Objectives	Teaching Aids
1. Comparison of units and Conversion of units	The student can express the relationships between units of measure appropriate to the grade level and can rename a measure in other units. For example: 1 kg weighs (more, less) than 500 grams 14 dm = _____m	

Major Topic	Behavioral Objectives	Teaching Aids
D. COMPUTATIONS WITH MEASURES		
1. Addition and Subtraction	The student can compute with measures appropriate to the grade level, assign the proper unit to the result, and rename if necessary. For example: 3 litres + 5 litres = 8 litres 50 cents - 38 cents = 12 cents = $.12	
E. SUMMARY See Appendix, Page 330		

NUMBER AND OPERATIONS GRADE FOUR

Major Topic		Behavioral Objectives	Teaching Aids
A. THE NATURAL NUMBERS COUNTING & NUMERATION			
1. Concepts Related to Counting			
1 - 1,000,000	(a)	Given a number, the student can identify the numbers which precede and follow it.	Refer to Grade III, Section A-1-(a).
1 - 1,000,000	(b)	Given a number greater than 1,000, the student can verbalize his notion of the quantity involved. For example: "A million dollars is having a thousand dollars in each of a thousand banks."	Display as many sheets as necessary on a wall to show 1,000,000 X's.
2. Counting the Number in a Set			
Collections	(a)	Given a description of a set, the student can distinguish between members of the set and things which are not members.	Refer to Grade III, Sec. A-2-(a).
One-to-one correspondence	(b)	Given two equivalent sets (objects or pictures), the student can demonstrate a one-to-one matching between members of the sets by constructing lines or by physically associating the objects.	Use set notation as a means of presenting problems for matching equivalent sets, e.g. A = {1, 3, 5, 7, 9} B = {□, △, ○, ○, ⬭}

Major Topic	Behavioral Objectives	Teaching Aids
One-to-many	(c) The student can describe familiar physical situations in which a one-to-many matching occurs.	For example: One notebook to three rings, or one cat to four paws.
Equivalent & non-equivalent sets	(d) Given two sets (objects, pictures, descriptions), the student can identify them as equivalent or non-equivalent.	Refer to Grade III, Sec. A-2-(d).
Empty Set	(e) The student can identify and describe sets which have no members.	For example: The set of all living purple tigers 200 years old or older has no members. Refer to Grade III, Sec. A-2-(d)
Subsets	(f) Given a set, the student can identify and describe a subset.	For example: Fords are a subset of the set of all automobiles.
Whole numbers 0-1,000,000	(g) The student can determine the cardinal number of a given set.	Refer to Grade III, Section A-2-(f) Refer to Grade III, Section A-2-(g)
3. Reading, Recognizing & Writing Numbers	(a) The student can identify, name, read, and write many different names for the same number.	For example: $12 = 7 + 5 = 27 - 15 = 3 \times 4 = 48 \div 4$
Names for Numbers	(b) The student can identify, name, read, and write numerals for whole numbers.	
Whole Numbers 0-1,000,000	(c) Given a numeral such as 467,304 the student can write the words: four hundred sixty-seven thousand, three hundred four.	

Major Topic	Behavioral Objectives	Teaching Aids
4. Using Ordinal Names		
Ordinal Numbers Beyond 20th.	(a) The student can write the symbols $<$, $>$, $=$, $\neq$ to express the relationship between two given numbers.	Refer to Grade III, Sec. A-4-(a)
	(b) Given a situation involving a number, the student can identify the number as cardinal or ordinal (or neither).	For example: I am fifth in line. (ordinal) I have five cars. (cardinal) I drive a Ford Galaxie 500. (Neither cardinal nor ordinal) Refer to Grade III, Sec. A-4-(c)
5. Place Value		
Place value - six digit numerals	Given a base ten (decimal) numeral, the student can write the expanded numeral in the following way: 473, 245 = (4 x 100,000) + (7 x 10,000) + (3 x 1000) + (2 x 100) + (4 x 10) + (5 x 1).	Stress that the value of each column is ten times the one to its right. For example: 10 x 10 10 x x 10 10 10 x x x 10 10 10 10 x x x x 10 10 10 10 10 1 4 7 3 2 4 5

Major Topic	Behavioral Objectives	Teaching Aids
6. **Non-decimal Numeration**		
Roman Numerals through L	(a) Given a numeral such as 52, the student can write the Roman numeral LII. (b) Given a Roman numeral such as LVI, the student can write the Arabic numeral 56.	Refer to Grade III, Sec. A-5-6-(a,b)
7. **Rounding & Estimation**		
Other notation Rounding to the nearest ten, hundred, thousand.	Given a numeral such as 4651, the student can round it to 4650 to the nearest ten, to 4700 to the nearest hundred, and to 5000 to the nearest thousand.	Use appropriate parts of a number line marked in units that will be convenient for the units to which it is desired to round the given number.

Major Topic	Behavioral Objectives	Teaching Aids
B. NATURAL NUMBERS OPERATIONS		
1. Addition & Subtraction		
Inverse relationship	(a) The student can solve equations such as: 207 + ___ = 896; 150 - ___ = 98; ___ + 27 = 4068; ___ - 2146 = 9763	Develop this concept by using smaller numbers and when necessary, have the students use the number line to find the answer. Stress subtraction as the inverse of addition. (Refer to previous grades for examples). Have the students estimate what the answer will be before actually solving the problem.
	(b) The student can check subtraction problems by addition.	
Basic facts through sums of 18	(c) Given any single digit addition or subtraction combination, the student can *immediately name the sum or difference.	
Algorithms Column addition & subtraction (5 digit numerals) with & without renaming	(d) The student can name the sums and differences for problems such as: $\begin{array}{r} 72{,}048 \\ +\ 19{,}969 \\ \hline \end{array}$ $\qquad$ $\begin{array}{r} 40{,}001 \\ -\ 28{,}973 \\ \hline \end{array}$	Where necessary, have the students write the problem in expanded notation, e.g. $\begin{array}{rl} 412 & - 4 \text{ hundreds} + 1 \text{ ten} + 2 \text{ ones} \\ -\ 235 & - 2 \text{ "} \quad + 3 \text{ "} + 5 \text{ "} \\ \hline \end{array}$ By regrouping in two steps, we would have: $\begin{array}{l} 3 \text{ hundreds} + 10 \text{ tens} + 12 \text{ ones} \\ -\ (2 \text{ "} \quad + 3 \text{ "} \quad + 5 \text{ "}) \\ \hline 1 \text{ hundred} + 7 \text{ tens} + 7 \text{ ones} \end{array}$ or 177

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*Immediately is defined as 5 seconds or less

Major Topic	Behavioral Objectives	Teaching Aids
1. Addition & Subtraction (continued)		
Other notation	(e) The student can identify and name sums, differences, missing addends, and missing operational signs in problems written in both horizontal and vertical notation.	
2. Properties of Natural Numbers		
Properties Commutative & associative properties of addition	(a) Given an addition problem with three to six addends, the student can demonstrate how to find the sum in the easiest way be renaming and rearranging the addends.	
Identity element for addition	(b) The student can solve equations such as: 409 + ___ = 409; 116 - ___ = 116 + ___; ___ - 3412 = 0; and ___ - 0 = 23,809.	
3. Multiplication & Division		
Definition	(a) Given a multiplication or division problem such as 7 x 9 = ___ or 54 ÷ 6 = ___, the student can design a simple experiment involving sets on a number line to determine and name the product or quotient.	Have available a number line, tongue depressors, poker chips, bottle caps, etc. or encourage students to demonstrate experiments with tally marks and write the mathematical statement for each experiment.

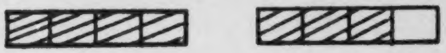
Major Topic	Behavioral Objectives	Teaching Aids
3. Multiplication & Division (continued)	(b) Given a multiplication or division problem such as 7 x 9 = ___ or 54 ÷ 6 = ___, the student can demonstrate how to find the product or quotient by repeated addition or subtraction.	For example: $3 \times 7 = 7 + 7 + 7$ 12 ÷ 4 = 3 : because $\begin{array}{r}12\\-4\\\hline 8\end{array} \xrightarrow{} \begin{array}{r}8\\-4\\\hline 4\end{array} \xrightarrow{} \begin{array}{r}4\\-4\\\hline 0\end{array}$
Inverse relationship	(c) Given a set model (rectangular array), the student can write two multiplication and two division equations to describe the physical situation.	For example: $4\left\{\overbrace{\begin{array}{cccccc}0&0&0&0&0&0\\0&0&0&0&0&0\\0&0&0&0&0&0\\0&0&0&0&0&0\end{array}}^{6}\right.$ DO $4 \times 6 = 24$; $6 \times 4 = 24$ UNDO $24 \div 4 = 6$; $24 \div 6 = 4$
Basic facts Through products of 81 (one digit)	(d) Given a multiplication or division combination such as 9 x 7 or 42 ÷ 6, the student can *immediately name the product or quotient and use sets or a number line to prove his result.	Use flash cards, number fact records, speed drills, and relay races to develop speed of response. Give some of the students an opportunity to record some problems on a tape recorder for the other students to solve.
	(e) The student can identify and name products, quotients, missing factors, and missing operational signs in problems written in both horizontal and vertical notation.	

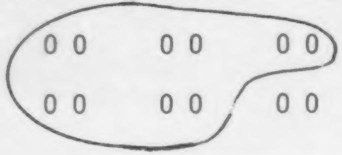
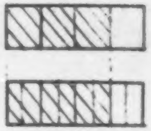
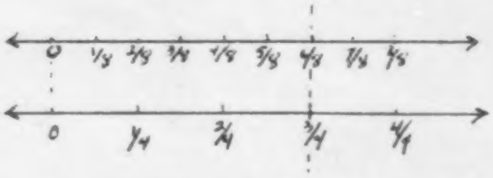
*Immediately is defined as 5 seconds or less

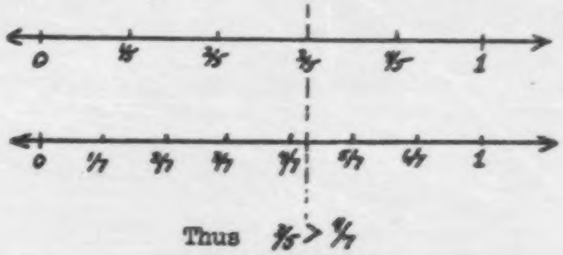
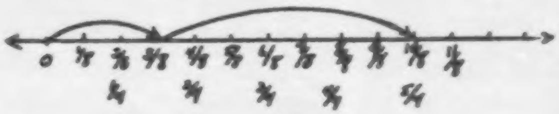
Major Topic	Behavioral Objectives	Teaching Aids
Algorithms Factors of 10, 100, & multiples of 10 & 100	(f) The student can name the products and quotients for problems such as: 1 x 23 = ___ 3 x 23 = ___ 10 x 23 = ___ 30 x 23 = ___ 100 x 23 = ___ 300 x 23 = ___ 4900 - 1 = ___ 4900 - 7 = ___	Refer to Grade III, Section B-2-(k)
Multiplication-vertical notation (4 digit factor by 2 digit factor) with & without regrouping	(g) The student can name the products for problems such as: $\begin{array}{r}21\\ \times\ 34\\ \hline\end{array}$ $\begin{array}{r}2976\\ \times\ 24\\ \hline\end{array}$ $\begin{array}{r}4005\\ \times\ 48\\ \hline\end{array}$	It is possible to approach this type of problem by first discussing the "do-it-to-both" rule, e.g. 3 x 16 = 3 (10 + 6) = (3 x 10) + (3 x 6) You can then develop the "double-do-it-to-both" rule, e.g. 5 x 7 = (3+2) x (6+1) = (3x6) + (3x1) + (2x6) + (2x1) = 18 + 3 + 12 + 2 = 35 THUS: 34 x 21 = (30 + 4) x (20 + 1) = (30 x 20) + (30 x 1) + (4 x 20) + (4 x 1) = 600 + 30 + 80 + 4 = 714 You can develop "double-do-it-to-all-three" rules, etc.
	Note: See objectives (m) and (n) for Grade III for examples of Algorithms.	Refer to Grade III, Section B-2-(1)

Major Topic	Behavioral Objectives	Teaching Aids
Long division (4 digit dividend by 2 digit divisor)	(h) The student can name the quotients and remainders for problems such as: $9\overline{)585}$ $14\overline{)1008}$ $98\overline{)4673}$ $21\overline{)6658}$	Refer to Grade III, Sec. B-3-(m)
4. Properties of Natural Numbers		
Properties Commutative property of multiplication	(a) The student can demonstrate by using sets or a number line that 9 x 6 = 6 x 9.	This property can be demonstrated by drawing a rectangular array on a transparency and then rotating the transparency, e.g. * * * * * * * * * * * * * * * * * * * * * * * * 3 x 4 = 12 4 x 3 = 12
	(b) The student can solve equations such as: 27 x ___ = 3 x 27 and 443 x ___ = ___ x 443.	
Associative property of multiplication	(c) The student can solve equations such as: $300 \times 27 = (3 \times \boxed{}) \times 27 =$ $3 \times (\boxed{} \times 27) =$ $3 \times (27 \times \boxed{}) =$ $(3 \times 27) \times \boxed{} =$ $81 \times \boxed{} =$	It is desirable for students to be aware of the fact that multiplication is associative and commutative and the advantages of using these properties, e.g. 300 x 27 = (3 x 100) x 27 = 3 x (100 x 27) = 3 x (27 x 100)=(3 x 27 x 100) = 81 x 100 = 8100
	NOTE: See objectives (m) and (n) in Grade III for examples of algorithms.	

Major Topic	Behavioral Objectives	Teaching Aids
Associative property of multiplication (continued)	(d) Given a problem, such as $7 \times 5 \times 8 = \underline{\quad}$, the student can indicate grouping of factors which will make the multiplication easiest by enclosing the 5 and 8 in parentheses.	For example: $7 \times (5 \times 8) = \underline{280}$
Identity element for multiplication	(e) The student can solve equations such as: $1 \times 453 = \underline{\quad}$; $32 \times \underline{\quad} = 32$; $24 - 27 = \underline{\quad}$; and $45 - \underline{\quad} = 45 \times \underline{\quad}$.	Refer to Grade III, Sec. B-4-(d)
Multiplicative property of 0	(f) The student can solve equations such as: $7320 \times 0 = \underline{\quad}$; $0 = \underline{\quad} \times 700$; $7 - 0 = \underline{\quad}$; $0 \times . = \underline{\quad}$; and $0 - 1 = \underline{\quad}$.	Refer to addendum for a discussion of the properties of zero.
Distributive property of multiplication over addition	(g) The student can demonstrate by using sets (rectangular arrays) that: $9 \times 7 = (5 \times 7) + (4 \times 7)$ $= 35 + 28$ $= 63$ and $9 \times 7 = (9 \times 3) + (9 \times 4)$ $= 27 + 36$ $= 63$	Refer to Grade III, Sec. B-4-(f)
	(h) The student can solve equations such as: $72 \times 1458 = (\square \times 1478) + (\triangle \times 1478)$ and $(40 \times 762) + (8 \times 762) = \underline{\quad} \times 762$.	Apply the distributive property in the way that will make the problem as easy as possible, e.g. $72 \times 1478 = (70 \times 1478) + (2 \times 1478)$ and $(40 \times 762) + (8 \times 762) = \underline{48} \times 762$.

Major Topic	Behavioral Objectives	Teaching Aids
C. RATIONAL NUMBERS 1. Reading & Writing the Fractional Numeral		
Rational Numbers Denominators of 1, 2, 3, 4, ...	(a) Given a model of three-ninths, the student can identify, name, read, and write a numeral (fraction) for the rational number associated with the model.	Refer to Grade III, Sec. C-1-(a).
	(b) Given a fraction such as 3/5, the student can identify, name and distinguish the numerator and the denominator.	Stress the fact that the denominator tells the number of parts into which the whole unit is divided and the numerator tells the number of those parts with which we are working.
Improper fractions & mixed numerals	(c) Given a model such as:  the student can identify, name, read, and write the fraction 7/4 and/or the mixed numeral 1 3/4 for the rational number associated with the model.	Stress the fact that 1 3/4 means $1 + \frac{3}{4} = \frac{4}{4} + \frac{3}{4} = \frac{4+3}{4} = \frac{7}{4}$ and the converse is true: $\frac{7}{4} = \frac{4+3}{4} = \frac{4}{4} + \frac{3}{4} = 1\ 3/4$
Rational Numbers Denominators of 1, 2, 3, 4, ...	(d) The student can construct and identify models for rational numbers.	Have available many different materials that can be used to construct these models such as tongue depressors, poker chips, egg cartons, play money, flannel board parts, etc.

Major Topic	Behavioral Objectives	Teaching Aids
Rational Numbers (continued)	(e) Given a set of 12 objects, the student can construct a model for 5/6 by grouping the members into subsets. For example:  	
2. Equivalent Fractions & Inequalities		
Equivalent fractions	(a) Given a fraction such as 7/8, the student can write a set of fractions which are equivalent to 7/8. For example: 7/8 = 14/16 = 21/24 = 28/32 = etc. (b) The student can rename a given fraction in simplest form. For example: 12/32 = 3/8	Use circles, rectangles, or the number line divided into fractional parts. For example ask student to select 3 of the 4 equal parts of one rectangle, and 6 of 8 equal parts of another rectangle the same size to show that 3/4 = 6/8.  

Major Topic	Behavioral Objectives	Teaching Aids
3. Ordering Fractions Order relations	Given two rational numbers such as 3/5 and 4/7, the student can name them with like fractions and write the symbols , , = , = to express the relationship between them.	Student may use two number lines, one marked in 5ths, the other in 7ths, to compare which fraction is the larger.  0 1/5 2/5 3/5 4/5 1 0 1/7 2/7 3/7 4/7 5/7 6/7 1 Thus 3/5 > 4/7
D. THE RATIONAL NUMBERS OPERATIONS		
1. Addition & Subtraction		
Definition (fractions with the same same denominators)	(a) Given a model (region or number line) for the addition or subtraction of two rational numbers, the student can determine and name the sum or difference. (See objective D-(a), Grade III, for examples of models).	Use the number line to show that: 3/8 + 7/8 = 10/8 = 1 2/8 = 1 1/4. Thus:  0 1/8 2/8 3/8 4/8 5/8 6/8 7/8 8/8 9/8 10/8 11/8 1/4 2/4 3/4 4/4 5/4

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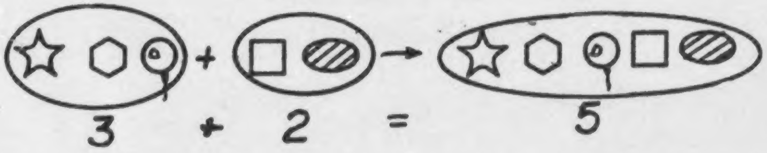
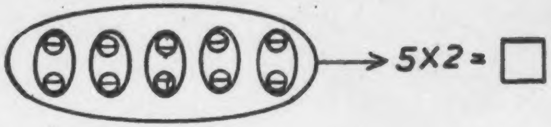
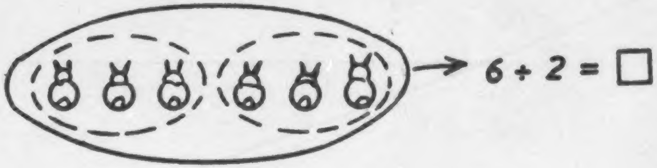
Major Topic	Behavioral Objectives	Teaching Aids
Addition & Subtraction (continued)	(b) Given an addition or subtraction problem such as 3/8 + 7/8 or 5/9 - 2/9 = ___, the student can name the sum or difference as a proper or improper fraction in lowest terms, or as a mixed numeral, and use regions or a number line to prove his result.	Permit student to work with 8ths of a "pie" cut out of paper circles to convince himself that 3/8 - 2/8 = 1/8, etc.
2. Operation with Decimals		
Algorithms Decimal notation (money)	The student can name the sums and differences for problems such as: $\begin{array}{r} \$\ \ 8.76 \\ 349.06 \\ \underline{+\ 23.55} \end{array} \qquad \begin{array}{r} \$\ 1001.00 \\ -\ 998.99 \\ \underline{\qquad\quad} \end{array}$	Permit student to actually handle play money to help him determine the correct answer to problems such as these. The student who understands and can work confidently with fractional notation may not find it necessary to develop separate rules to work with decimals. He need only convert his decimals to common fractions, perform the indicated operation, and convert back to decimal notation. Thus: .2 + .37 = 2/10 + 37/100 = 20/100 + 37/100 = 57/100 =.57 This type of conversion experience should be shared by all students at least at first in the teaching of decimals.

Major Topic	Behavioral Objectives	Teaching Aids
E. THE INTEGERS		
1. The Negative Rationals	Given $^-\frac{1}{4}$, $^-\frac{1}{2}$, $^-\frac{3}{4}$, $^-\frac{5}{4}$, $^-\frac{3}{2}$, $^-\frac{7}{4}$, ... the student must be able to correctly locate on a number line the points corresponding to the numbers.	Use a number line of sufficient size so that it can be easily marked into halves, fourths, eighths, etc. for the student to find important fractions on the line. $^-1$ ... 0
2. Addition & Subtraction of integers	(a) Given a set of "one-digit" integers having less than two digits, the student must be able to correctly determine the difference of any two. (b) Given a set of integers having less than two digits; the student must be able to correctly determine the difference of any two.	See Postman Story in addenda, page 307. The teacher will also find it useful to permit students to use the number line to find sums and differences. For example: $^+5 - {}^-2$ means to find the difference (or distance) on the number line from $^-2$ to $^+5$. As student counts units, he discovers that distance is 7; thus $5 - {}^-2 = 7$.
3 Multiplication & Division of Integers	(a) Given a set of integers having less than two digits, the student must be able to correctly determine the product of any two. (b) Given a set of integers having less than two digits, the student must be able to correctly determine the quotient of any two that are evenly divisible.	See Postman Story in addenda for justification of rules of signs in multiplying signed numbers. Student may use frame arithmetic to develop division facts once he has mastered multiplication facts. Thus, to find $^-15 - {}^-3$, he must find a number to fill the "box" so that $^-3 \times \square = {}^-15$. If the student can discover (by trial and error, if necessary) that a $^+5$ is required in the "box", then it follows that because $^-3 \times {}^+5 = {}^-15$, then $^-15 - {}^-3 = {}^+5$. Similarly, other facts for the various combinations of positive and negative numbers can be established.

SETS, FUNCTIONS, RELATIONS, SYSTEMS, AND LOGIC

GRADE FOUR

Major Topic	Behavioral Objectives	Teaching Aids
A. SETS		
1. Collections	Given a description of a set, the student can distinguish between members of the set and things which are not members.	Use flannel board with collections of small objects and pictures. Example: The set of letters in the word "play". $\{p, l, a, y\}$
2. One-to-one correspondence	Given two equivalent sets (objects or pictures), the student can demonstrate a one-to-one matching between members of the sets by constructing lines or by physically associating the objects.	See Grade Three, Section A-2-(a)
3. One-to-many correspondence	The student can describe familiar physical situations in which a one-to-many matching occurs. For example: One notebook to three rings, or one cat to four paws.	
4. Equivalent & non-equivalent sets	(a) Given two sets (objects, pictures, descriptions), the student can identify them as equivalent or non-equivalent.	If Set T = $\{A, B, C\}$, Set R = $\{D, E, F\}$, and Set W = $\{G, H, I, J\}$ then Set T is equivalent to Set R, but not to Set W. If two sets have the same number of elements, the sets are equivalent.
Empty Set	(b) The student can identify and describe sets which have no members such as the set of all live elephants in the classroom.	
Subsets	(c) Given a set, the student can identify and describe a subset.	For example: Given the set of letters in the alphabet, the student can give a subset such as the vowels.

Major Topic		Behavioral Objectives	Teaching Aids
5. Cardinal Number 0 - 1,000,000	(a)	The student can determine the cardinal a given set.	
6. Set Union	(a)	Given two sets, the student shall be able to join the two sets and get a third set.	See Grade Three, Sect. A-6(a). Also see illustration below. 
	(b)	Given a set of elements, the student shall be able to remove a set and get a remaining set.	See Grade Three, Sect. A-6 (b).
7. Commutative, Associative, & Distributive property		The student shall be able to demonstrate the commutative, associative, and distributive properties through the use of sets.	See Grade Three, Sections A-7 (a), (b), and (c).
8. Multiplication	(a)	The student can describe set union as multiplication.	
Set Partitioning	(b)	The student shall be able to describe a set partitioning as division.	

<table>
<tr><th>Major Topic</th><th>Behavioral Objectives</th><th>Teaching Aids</th></tr>
<tr><td>n(A)</td><td>(c) The student shall be able to associate a number with a set that names the number of elements in the set either by using set notation or simply naming the number of elements of the set.</td><td>A = [□, △, ▱, ⏢, ○, ⬭]
N(A) = 6</td></tr>
<tr><td>B. RELATIONS & FUNCTIONS</td><td></td><td></td></tr>
<tr><td>1. "What's My Rule?"</td><td>Given the game situation described in the addenda, the student will be able to determine the missing values, and state the rule of the game.</td><td>
<table>
<tr><th>In</th><th>Out</th></tr>
<tr><td>1</td><td>4</td></tr>
<tr><td>2</td><td>5</td></tr>
<tr><td>3</td><td>6</td></tr>
<tr><td>4</td><td>?</td></tr>
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<tr><td>1</td><td>2</td></tr>
<tr><td>3</td><td>4</td></tr>
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<table>
<tr><th>In</th><th>Out</th></tr>
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<tr><td>2</td><td>6</td></tr>
<tr><td>3</td><td>11</td></tr>
<tr><td>4</td><td>18</td></tr>
<tr><td>5</td><td>?</td></tr>
</table>
</td></tr>
<tr><td>2. Number Patterns</td><td>The student shall be able to complete the next three elements of the following sets.
A = { 2, 4, 6, ___, ___, ___, . . . }
B = { 4, 9, 14, ___, ___, ___, . . . }</td><td>See addenda.</td></tr>
</table>

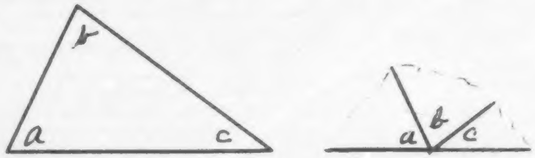
Major Topic	Behavioral Objectives	Teaching Aids
9. Equal Sets	Given two sets, the student can identify them as equal or not equal.	Set A= $\{1, 2, 3\}$ Set B= $\{3, 1, 2\}$ Set A = Set B Set C= $\{2, 3, 4\}$ Set D= $\{2, 4, 6\}$ Set C $\neq$ Set D If Set A has exactly the same elements as Set B, though not necessarily listed in the same order, then Set A = Set B. If two sets have the same number of elements, they are equivalent sets but not necessarily equal.
10. Set Symbolism	(a) Given a description of Set A and a larger set containing Set A, the student can select Set A from the larger set.	For example: From the set of students in the classroom, select the set of students whose names begin with "C". See Grade Three, Sect. A-9 (a).
	(b) Given a description of a set, the student shall be able to form the set using set symbolism.	For example: The set containing the letters of the word "dog". $\{d, o, g\}$
	(c) Given two sets, the student can represent the union and intersection of the two sets in set notation.	$A = \{a, b, c, d, e\}$ $B = \{c, a, t\}$ $A \quad B = \{a, b, c, d, e, t\}$ Note: In sets elements are listed only once, thus in $A \cup B$ above there is only one "a" and one "c". $A \cap B = a, c$

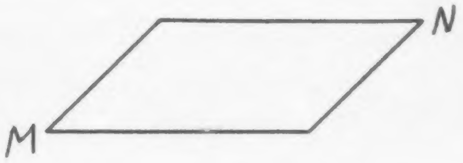
Major Topic	Behavioral Objectives	Teaching Aids
C. MATHEMATICAL SENTENCES		
1. Open, true or false sentences	(a) The student shall be able to classify the following types of sentences as open, true or false, and answer or correct the open and false sentences. $7 + \square = 9$, $9 - 7 = 3$, $3 \times 4 = 12$, $\triangle - 3 = 4$	
	(b) The student shall be able to use the "is not equal", "less than" and "greater than" signs in mathematical sentences.	$8 = 4 + 2$, $9 >$ ⬡, $7 < 4$
	(c) The student shall be able to use parentheses to show order in mathematical sentences involving addition, subtraction, multiplication, and division.	$(4 + 3) + (2 \times 4) - (6 \div 3) =$ $(7 + 8) - 2 = 15 - 2 = 13$
	(d) The student can find the solution set to Mathematical sentences with two different variables.	$\square + \square = 10$; $\square + \triangle = 18$ $6 \times 5 = (X \times 5) + (Y \times 5)$
	(e) The student can translate an English sentence into a mathematical sentence and solve.	For example: A boys' club with eight members bought a flag for $5.36. If they shared the cost equally, how much did each boy pay? $\$5.36 \div 8 = \square$

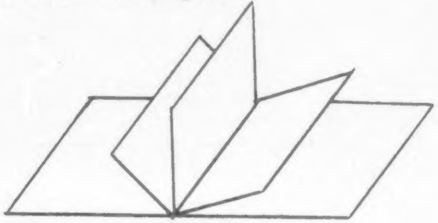
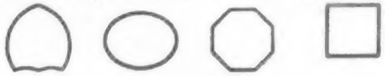
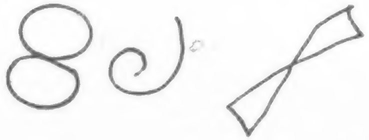
GEOMETRY

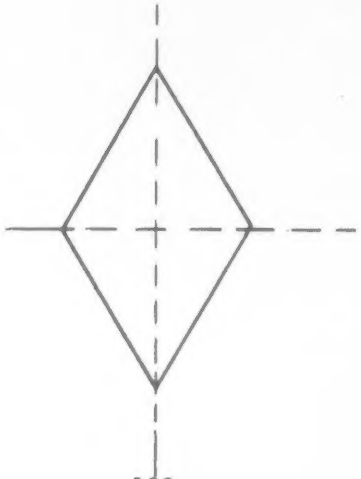
GRADE FOUR

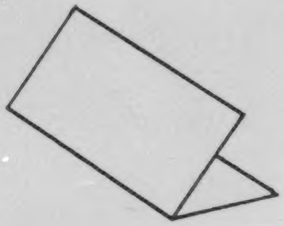
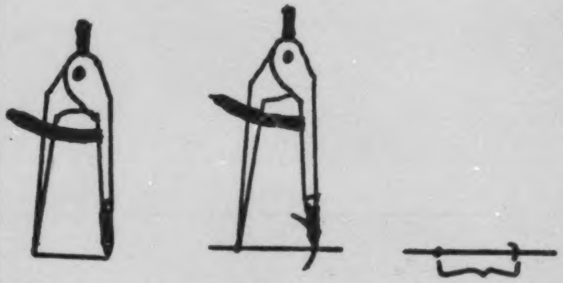
Major Topic	Behavioral Objectives	Teaching Aids
A. RECOGNITION & DESCRIPTION OF PLANE FIGURES		
1. Plane Figures (as sets of points) Point	(a) Given models of the plane geometric figures named on the left (wire, paper or flannel cutouts, pencil or chalk outlines), the student can *identify, name,* and *distinguish* among them.	
Path (curve) Line Line segment Ray Angle Right Angle Polygon Triangle Right triangle	(b) Given the task of *recognizing* right angles, the student will *make* a model, *apply* it, and *interpret* the results, indicating which angles are right angles.	Having made a model of a right angle by folding a rough piece of paper carefully, the student checks his environment & reports those objects he finds which fulfill or embody the principal of the right angle. Using their models of right angles, students measure corners of table tops, where the walls meet, the relationship of desk leg and floor, or blackboard, picture, and window corners, etc.
Quadrilateral (diagonal) Parallelogram Square Rectangle	(c) Given a set of triangles, the student can select those which are isosceles and equilateral triangles.	Isosceles triangles have two sides of equal length. Equilateral triangles have three sides of equal length. Parallelograms have opposite sides parallel and of equal length. Isosceles triangles are readily made by folding or cutting various sized squares along the diagonals. Any large paper model of a triangle can be folded to check the length of any one side against another. Triangles thought to be isosceles may be similarily tested.
Circle Center Radius Diameter Chord	(d) Given a set of geometric figures, the student can identify those that are parallelograms, squares and rectangles.	

Major Topic	Behavioral Objectives	Teaching Aids
1. Plane Figures (continued)	(e) Given cutouts of triangles, the student will *discover the relation* of the sum of the angles to a right angle.	The corners of the triangles may be torn off and fitted as below:  and measured by using a model of a right angle. Applying it twice, the student finds the sum of the angles equals two right angles.
	(f) Given the assignment to find a triangle whose angles do not sum to two right angles, the student will discover a generalization that probably all triangles sum to two right angles.	
B. RECOGNITION & DESCRIPTION OF SOLID FIGURES		
1. Space Figures Right Rectangular prism Cube Sphere Cylinder Cone Pyramid	Given models of the space figures named on the left (wood or plastic solids, paper models, sketches, etc.), the child can *identify*, *name*, and *distinguish* among them.	

Major Topic	Behavioral Objectives	Teaching Aids
C. CONCEPT OF POINT, LINE SETS OF POINTS		
1. Sets of points	(a) The student can give a verbal description of space as the *set of all points*, using any visual aids he wished to strengthen his communication.	
	(b) The student can give a verbal description of a plane as a flat surface which contains lines and points.	A plane is a particular set of points, and, like points and line, it has location only in space. A plane may be depicted as shown below. The picture, itself, may not suggest that a plane extends outward indefinitely but it must be considered as doing so.  A physical horizontal flat surface, as a flat board held in many positions, provides an intuitive development for the concept of a plane. Remove the board from view and let the students imagine the space that the board occupied; then have them imagine the *set of points* extended outward in all directions.

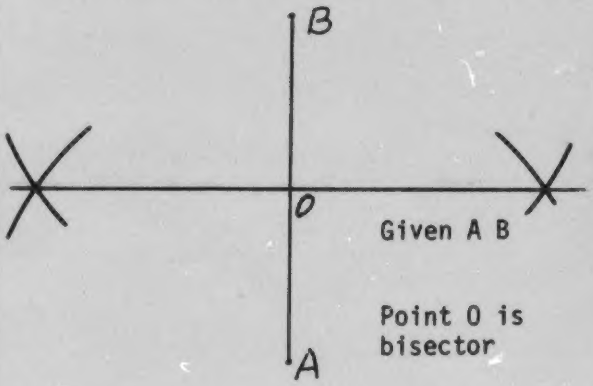
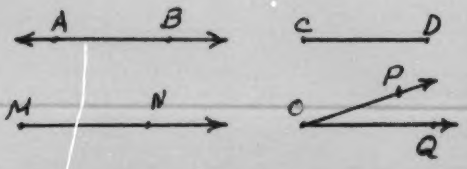
Major Topic	Behavioral Objectives	Teaching Aids
1. Sets of points (continued)	(c) The student can give a verbal description of lines as *intersections of planes*.	Two *intersecting planes* establish a line. Many planes could also intersect in one line. The spine of an open book can be thought of as a line intersected by the many planes which are the pages. 
	(d) The student can give a verbal description of a *circle as the set of all points* in a plane that is at the same distance from a fixed point.	
	(e) The student can give a verbal description of parallel lines in a plane which do *not* intersect.	
D. CLOSED, OPEN CURVES AND REGIONS		
1. Simple Closed Curves	Given a set of plane figures, the child can *distinguish* the simple closed curves and *identify* the *interior* and *exterior* regions.	Every geometric figure in a plane, which ends where it begins, and which separates the plane into *only two* regions, is a *simple closed curve.* Example: Some simple closed curves are:  These figures are *not* simple closed curves: 

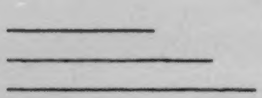
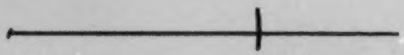
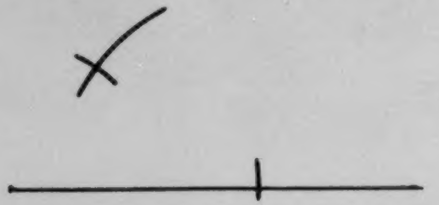
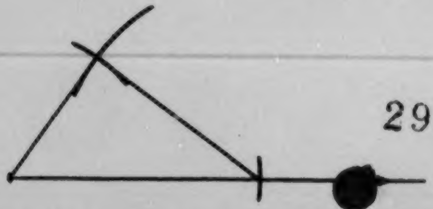
Major Topic	Behavioral Objectives	Teaching Aids
E. PROPERTIES OF TWO- AND THREE-DIMENSIONAL FIGURES		
1. Volume	The child can demonstrate his understanding of the concept of volume by filling the interior space of a right rectangular prism (i.e., chalkbox, shoebox, etc.) with unit cubes.	
2. Congruence	(a) The student can *match* congruent angles using models or tracings. (b) The student can *construct* congruent figures on the geoboard.	
3. Symmetry	(a) The student can *demonstrate* that some figures have two or more axes of symmetry through paper folding. For example, two axes of symmetry are indicated for the figure shown below: 	Designs involving a particular number of squares of graph paper may be analyzed to determine which possesses symmetry.

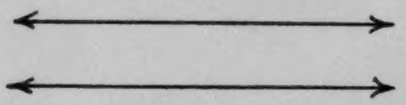
Major Topic	Behavioral Objectives	Teaching Aids
3. Symmetry (continued)	(b) The student will *demonstrate* that all congruent figures are similar, but not all similar figures are congruent.	Similar geometric figures have the same shape but not necessarily the same size. Congruent geometric figures are exactly alike in every respect. All squares are similar while any two whose dimensions differ are *not* congruent. All figures whose angles or segments fit each other exactly are both congruent and similar.
F. CONSTRUCTION -- APPROXIMATE AND EXACT 1. Construction	(a) The pupil can demonstrate through paper folding an understanding of a line as an intersection of two planes.	Intersection of two planes forms line: Folded paper 
	(b) The pupil can reproduce a line segment by using a compass and straightedge.	 Measure given segment scribed on new line reproduced line segment

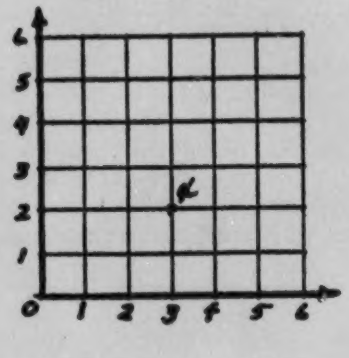
GEOMETRY

GRADE FOUR

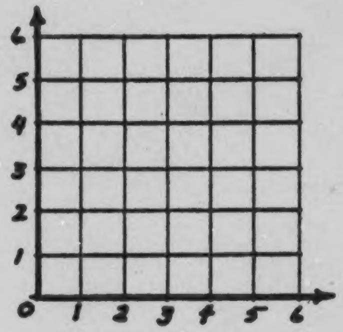
Major Topic	Behavioral Objectives	Teaching Aids
	(c) The student can *bisect* a line segment by using a compass and straightedge.	 Given A B Point O is bisector
	(d) The child can *draw* and *name* a set of points satisfying given conditions. For example: The set of all points in a plane, one cm from a given point, is a (*n*) __________.	Circle
	The set of all points, contained in two rays with a common endpoint, is a (*n*) __________.	Angle
	(e) The pupil can make *rough* pencil and/or chalk drawings (outlines) of the plane figures named under GEOMETRIC FIGURES.	
2. Plane Geometric Figures Line Line Segment Ray	(a) Using a straightedge, the pupil can construct models for lines, line segments, rays, angles, and quadrilaterals and label them.	

Major Topic	Behavioral Objectives	Teaching Aids
2. Plane Geometric Figures (continued)		
Right Triangle	(b) Using a straightedge and folded paper, the pupil can construct a right angle and a right triangle.	The children may be asked to check the *relation of diagonals* of a figure to determine which diagnonals meet as right angles.
Triangle	(c) Given three sides of a triangle and using a straightedge and compass, the pupil can construct the triangle.	Given:  *Side Three* *Side Two* *Side One* reproduces side one  Scribe from ends of *side one* the length for *sides two and three.*  Connect the points and *complete the triangle.* 

Major Topic	Behavioral Objectives	Teaching Aids
	(d) Given the center and radius and using a string or a compass, the student can construct a circle.	Some authors propose delaying the use and introduction of the compass as an instrument to make a circle because it obscures the most important property of the circle, that its radii are all equal. First attempts to draw a circle should be their own such as tracing a plate or jar lid, etc. Having cut out a circle, children may be asked to find the center. They will probably begin folding the circle which proves to be not only a good way to find the center but also reveals the symmetry of a circle when it's folded about its diameter.
3. Properties of line Parallel lines	(a) The pupil can sketch and describe parellel lines.	For example: Parallel lines are lines in the same plane that never meet.  The student could be required to point out in the classroom examples of parallel lines; i.e., the opposite sides of a table, the lines made by the wall meeting the ceiling and floor, etc.

Major Topic	Behavioral Objectives	Teaching Aids
3. Properties of Line (continued)		
Intersecting Lines	(b) The pupil can sketch and describe intersecting lines.	For example: Intersecting lines are lines that have one point in common.
G. COORDINATE SYSTEMS AND GRAPHS		
	(a) The student can explain and demonstrate that a line segment is a set of points.	Establish two number lines at right angles, with zero at the intersect and each line similarly divided and divisions labelled as in the figure below.
	(b) The student can explain and demonstrate that points in a plane (the first quadrant) can be represented by (ordered) pairs of numbers (coordinates).	For example: point x is 3 units to the right of 0 and 2 above. 

GEOMETRY **GRADE FOUR**

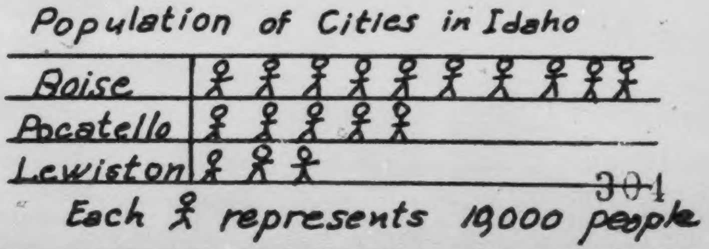
Major Topic	Behavioral Objectives	Teaching Aids
G. COORDINATE SYSTEMS AND GRAPHS (continued)		When lines parallel to the axes and coinciding with labeled points are drawn, the resulting point of intersection can be identified.  6 5 4 3 2 1 0 1 2 3 4 5 6 Once the concept of numbers and their correspondence to a set of points in the number line is established, the concept of pairing whole number with whole number may be developed. There are many (an unlimited number if the segments are extended indefinitely) ordered pairs which can be formed by matching the whole numbers. For example: (3, 4), (5, 2), (0, 6) are ordered pairs. In an ordered pair such as (3, 4), the number 3 is called the first component of the ordered pair, and the number 4 is called the second

Major Topic	Behavioral Objectives	Teaching Aids
G. COORDINATE SYSTEMS AND GRAPHS (continued)		Each of the first numbers of the ordered pairs corresponds to a point in the horizontal number line (X axis) and each of the second members corresponds to a point in the vertical number line (Y axies). Using these coordinatized lines, we can locate points in a quadrant of its plane that are named by ordered pairs.
	(c) The student can explain and demonstrate the use of the number line to illustrate division problems. See Grade III.	
H. METRIC		
1. Length	(a) Student will make measures of objects or distances, recognize and explain the differences attained by different students.	Try measuring the width of the room using a pencil as the unit of measure. There will be different measures even using the same pencil. This should reinforce the idea that measurement is always approximate.
	(b) Students will explain the greatest possible error that may occur when using various measuring devices and units.	
2. Area	The student can determine the effect of doubling, tripling, etc. the dimensions of a square or rectangle.	A graph or outline picture solution may be employed both to reveal and report results. Graph paper may be outlined.

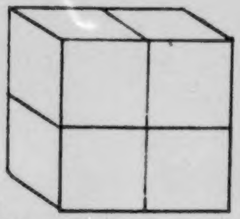
MEASUREMENT AND ESTIMATION

GRADE FOUR

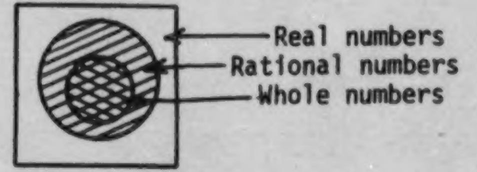
Major Topic	Behavioral Objectives	Teaching Aids
A. CONCEPT OF MEASUREMENT		
1. Process of Measuring	(a) Given a measurable physical property such as length, area, weight, temperature, etc., the student can select a suitable unit and/or measuring device and measure the property. For example: Weight can be measured by using a(n) _______________.	Tell the story of the origin of some of our measures. Then let students measure objects using some of these units of measure and compare the results with those obtained by using a standard unit of measure.
Arbitrary selection of unit	(b) The student can name at least two units suitable for naming the measure of a given physical property. For example: The area of a floor can be expressed in square metres or in square vinyl tiles.	Use laboratory material. See addendix.
Approximate nature of measurement	(c) The student can demonstrate his understanding of the approximate nature of measurement by stating the precision of the measure. For example: The distance between Moscow and Lewiston is 50 km correct to the nearest 5 km.	Use problems set up for laboratory material. Use road maps.
2. Concept of Graphs	The student understands that one picture represents more than one item. Each picture represents _____ items.	Have a collection of graphs that the student can use in answering questions.



Major Topic	Behavioral Objectives	Teaching Aids
B. MEASUREMENT OF PHYSICAL PROPERTIES		
1. Time Second, minute, hour	Using a clock, the student can read and write time to the nearest second and indicate A.M. or P.M.	Use a large cardboard clock for review of telling time. Provide problems requiring the amount of time it takes to travel from one place to another. (Involving both hours and minutes.)
2. Money	The student can perform computations involving money.	Discuss a shopping trip, pick a lunch. How much money have I? See addenda.
3. Linear Measurement Length Centimeter, meter kilometer	(a) The student can use various measuring devices (ruler, meter stick) to measure length in whole and fractional parts of units.	Follow up the discussion of kinds of units by giving an identical box, to each of two groups of children. Also give them two different sets of units, (one group a four cm piece of string, a plastic tile, and a box of alphabet blocks; the other group a six cm piece of stiff wire, a square bottom of a milk carton, and a box of one cm cubes.) Then give each a set of objects to measure.
Perimeter	(b) The student can determine and name the perimeter of a given polygon by adding the measures of the sides.	Use a geo-board, peg board, dot paper, with figures drawn on them.
4. Area Square units related to units of length	Given the measure of the base and altitude of a rectangular region in whole units, the student can determine and name the area of the region by multiplication (as the necessary multiplication facts and algorithms are learned.)	Try geo-board, checkerboard grid and clear plastic cut outs.

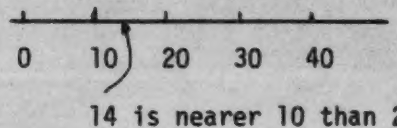
Major Topic	Behavioral Objectives	Teaching Aids
5. **Volume** Cubic units related to units of length	(a) Given a sketch of a right rectangular space figure, the student can determine and name the volume of the figure by counting the number of volume units, (cubes) needed to fill the space.	Use cubic cm blocks, liquid measuring devices, sand. Sugar cubes can be glued together and used as models. 
Liquid measure 250 ml, 500 ml, liter	(b) The student can name the standard metric units of liquid measure and measure the capacity of a given container to the nearest whole unit.	Use liquid measuring devices and containers of different sizes and shapes that will hold liquids.
6. **Weight-Mass** Gram, kilogram, metric ton	Using a scale, the student can measure the weight (mass), of a given object in whole and fractional parts of units. (Note: Students can name the metric ton as a unit of weight but are not expected to weight objects that heavy.)	Try guessing and checking. See addenda.
7. **Temperature** Celsius degrees	(a) Using a thermometer calibrated in celsius degrees, the student can read the temperature to the nearest degree.	Make a demonstration model of a thermometer. Your classroom should be equipped with an indoor and outdoor thermometer. Try to obtain other kinds of thermometers for study, a clinical thermometer, a meat thermometer. Test the boiling point of water with a candy thermometer.
8. **Graphs** Bar, pictograph, circle	(a) The student can read and use graphs in finding solutions to problems.	Have students bring graphs from magazines and newspapers.
	(b) The student can make bar and pictographs from given data or from data that they have collected.	Let them make up problems about their graphs.

Major Topic	Behavioral Objectives	Teaching Aids
Statistics Range Average	(c) The student can find the average of a series of numbers. (d) From a graph, the student can determine the range of the values.	Have students keep a record of the temperatures for one week. Let them determine the range of the temperatures and the average temperature for the week. Use other problems similar to the one above that uses measurements as well as finding averages.
C. RENAMING MEASURES		
1. Comparison of units and Conversion of units	The student can express the relationships between units of measure in other units. For example: 1 meter is (longer, shorter) than 90 cm	Provide meter stick and rulers for your students to use.
D. COMPUTATIONS WITH MEASURES		
1. Addition, Subtraction, Multiplication	The student can compute with measures appropriate to the grade level. 2 meters 25 centimeters -1 meter 75 centimeters translates to 225 cm -175 cm 50 cm or .5 m	

Major Topic	Behavioral Objectives	Teaching Aids
A. THE NATURAL NUMBERS COUNTING & NUMERATION 1. Counting the Number in a Set		
Sets Finite Infinite Empty	(a) The student can construct a Venn diagram to show the relationship of whole numbers to rational numbers.	Large examples of Venn diagrams on display would be helpful. Example: 
	(b) The student can classify a given set as: *Finite* -- has a definite number of members. *Infinite* -- has an unlimited number of members. *Empty* -- has no members.	Have the student give examples of each type of set. Examples: *Finite* $\{0, 1, 2, 3\}$ *Infinite* $\{0, 1, 2, 3, \ldots\}$ *Empty* {Purple elephants in Idaho}
Whole numbers 0 - 1,000,000,000	(c) The student can determine the cardinal number of a given set.	Examples: $0 = n\{\ \}$ $1 = n\{0\}$ $2 = n\{0, 1\}$

Major Topic	Behavioral Objectives	Teaching Aids
2. Reading, Recognizing, and Writing Numbers		
Names for Numbers	(a) The student can identify, name, read and write many different names for the same number.	For example: 6 7/10 = 6 + 7/10 = 6.7 = 6 7/10 = 2 + 4 + .7
Whole numbers 0 - 1,000,000,000	(b) The student can identify, name read and write numerals for whole numbers.	
	(c) Given a numeral such as 467,304, the student can write the words: four hundred sixty-seven thousand, three hundred four.	
	(d) Given a verbal phrase naming a number, the student can write the number words and/or the Arabic numeral.	
3. Using Ordinal Names		
Order relations	(a) Given a set of whole numbers, the student can write them in order from least to greatest and vice versa.	
	(b) The student can write the symbols <, >, =, and ≠ to express the relationship between numbers.	

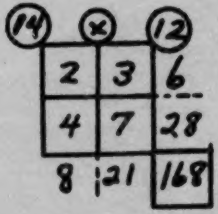
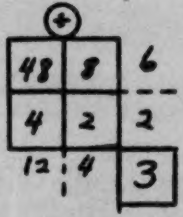
Major Topic	Behavioral Objectives	Teaching Aids
4. Place Value		
Place value-	(a) Given a base ten (decimal) numeral, the student can identify and name the place value for each digit.	For example: 473 millions / 231 thousands / 715 ones
	(b) Given a nine-digit numeral, the student can identify and name the *period* value for each group of three digits.	473,231,715 = (4 x 100,000,000) + (7 x 10,000,000) + (3 x 1,000,000) + (2 x 100,000) + (3 x 10,000) + (1 x 1,000) + (7 x 100) + (1 x 10) + (5 x 1).
Expanded notation-nine-digit numerals	(c) Given a numeral such as 473,231,715, the student can write the expanded numeral.	
	(d) Given a numeral such as 243,401,715, the student can write the expanded numeral illustrating period (rather than place) value.	273,401,715 = (273 x 1,000,000) + (401 x 1,000) + (715 x 1).
5. Non-decimal Numeration		
Roman numerals through M	(a) Given a numeral such as 104, the student can write the Roman numeral CIV.	A display chart of values of the Roman numeral symbols should be in the classroom.
	(b) Given a Roman numeral such as CII, the student can write the Arabic numeral 102.	Have the students find examples of the use of Roman numerals outside the classroom.

Major Topic	Behavioral Objectives	Teaching Aids
	(c) Given two numerals, one a Roman numeral and the other an Arabic numeral, the student can illustrate the difference between the two systems.	In the Roman numeral the "X" always has the value of ten, but in the decimal numeral the value of the "3" depends upon its *place* in the numeral. Therefore, XXX = 10 + 10 + 10, but 333 = 300 + 30 + 3.
Non-decimal numeration --base two, four, five	(d) Given a set with from 1 to 100 members, the student can group the members and write a base five numeral for the cardinal number of the set.	Use sets of tongue depressors for the student to group to illustrate base five. It is not necessary for students to become proficient in using the four operations and non-decimal numbers.
6. Rounding and Estimation		
Other notation Rounding to the nearest one, ten, hundred, thousand, tenth, hundredth	Given a numeral such as 14.36, the student can round it to 10 to the nearest ten, to 14 to the nearest one (whole number) and to 14.4 to the nearest tenth.	Examples from the monetary system are useful to illustrate rounding. Students may find it helpful to count by rounding group (i.e., 10, 20, 30 ...). Use a number line to indicate the nearest rounding group.
7. Primes and Composites		Example:  14 is nearer 10 than 20.
Prime & composite numbers	(a) Given a whole number greater than 1 and less than 100, the student can classify the number as: *Prime* - has exactly two factors *Composite* - has more than two factors	

Major Topic	Behavioral Objectives	Teaching Aids
Prime factorization	(b) Given a numeral for a composite number, the student can write a numeral which names the number as the product of prime numbers. For example: 60 = 2 x 2 x 3 x 5	Using the list of prime numbers as possible divisors some students may find the following method of complete factorization helpful. For example: $\begin{array}{r} 2\underline{)870} \\ 3\underline{)435} \\ 5\underline{)145} \\ 29 \end{array}$ $\therefore$ 870 = 2 x 3 x 5 x 29 Students should be let to discover that they need to try only the prime numbers as divisors that are less than the square root of the number to be factored. The student should list the primes from 2 through 97 2,3,5,7,11...,97
8. Exponential Notation	Given a product of Identical Factors, the student can write the product in exponential notation. Example: $2 \times 2 \times 2 = 2^3$	Illustrate that the exponent indicates the number of times a base is used as a factor. Example: 3^5 means 3 is used as a factor 5 times. Therefore: $3^5 = 3 \times 3 \times 3 \times 3 \times 3$ $3^5 = 243$ Emphasize 3^5 is not the same as 3 x 5.

Major Topic	Behavioral Objectives	Teaching Aids
B. THE NATURAL NUMBERS OPERATIONS		
1. Addition and Subtraction		
Inverse relationship	(a) The student can solve addition or subtraction problems with missing sums, differences, or addends. (b) The student can check subtraction problems by addition.	Example: ____ + 4687 = 10,000 and ____ - 23,459 = 1461
Basic facts through sums of 18	(c) Given only single-digit addition or subtraction combinations, the student can *immediately name the sum or difference.	
Algorithms Column addition and subtraction (nine digit numerals)	(d) The student can name the sums and differences for problems such as: 478,305,077 600,703,200 +23,989,196 - 5,876,999	
Other notation	(e) The student can identify and name sums and differences, missing addends, missing digits, and missing operational signs in problems written in both horizontal and vertical notation.	

Immediately is defined as 5 seconds or less.

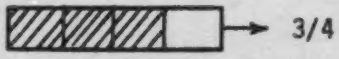
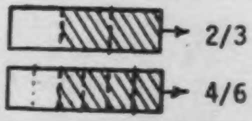
Major Topic	Behavioral Objectives	Teaching Aids
2. Properties of the Natural Numbers		
Properties Commutative & associative properties of addition	(a) Given an addition problem with three or more addends, the student can demonstrate how to find the sum in the easiest way by renaming and rearranging the addends.	Example: 3 + 5 + 7 = 3 + (5 + 7) = 3 + (7 + 5) = (3 + 7) + 5 = 10 + 5 = 15
Identity element for addition	(b) The student can solve equations such as 3905 + _____ = 3905; 5477 - 5477 = 5477 + _____; 0 = _____ - 380,357; and 8,777,300 + 0 = _____.	
3. Multiplication & Division		
Inverse relationship	(a) Given a multiplication equation such as 5 x 34 = 170, the student can write the two related division equations, 170 - 34 = 5 and 170 - 5 = 34.	Supplement by games, records, etc. See addenda. Try cross-number puzzles such as:
	(b) Given a multiplication problem with a missing factor such as _____ x 17 = 391, the student can write the related division equation, 391 - 17 = _____.	 14 × 12 2 3 6 4 7 28 8 21 168 
	(c) The student can check division problems by multiplication.	Leave out all of the numbers but four and let your students fill in the blanks for drill.

Major Topic	Behavioral Objectives	Teaching Aids
Basic facts Through products of 81	(d) Given any single-digit multiplication or division combination, the student can immediately* name the product or quotient and use sets or a number line to prove his result.	
Algorithms Multiplication - vertical notation (four-digit factor by three-digit factor)	(e) The student can name the products for problems such as: 9607 x 173 7956 x 789	
Long division (five-digit dividend by two-digit divisor)	(f) The student can name the quotients and remainders for problems such as: 47) 43,596 75) 10,000	Introduce the division of numbers by illustrating with successive subtraction. Refer to Grade Three, Section B-3-(m).
Short division (five-digit dividend by one-digit divisor) with and without remainder	(g) The student can use the short algorithm (short division form) to determine and name the quotients and remainders for problems such as: 9) 4203 5) 37,416	
Other notation	(h) The student can identify and name products, quotients, missing factors, and missing operational signs in problems written in both horizontal and vertical notation.	

*Immediately is defined as 5 seconds or less.

Major Topic	Behavioral Objectives	Teaching Aids
4. Properties of Natural Numbers		
	(a) Given a multiplication such as 468 x 1003 = _____, the student can select the vertical algorithm which will make the multiplication easier. For example: $\begin{array}{r}468\\ \times\ 1003\\ \hline\end{array}$ instead of $\begin{array}{r}1003\\ \times\ 468\\ \hline\end{array}$	
	(b) The student can check multiplication problems by reversing the order of the factors and multiplying again.	
Associative property of multiplication	(c) Given a problem such as 67 x 25 x 4 = _____, the student can indicate the grouping of factors which will make the multiplication easiest by enclosing the 25 and 4 in parentheses.	67 x (25 x 4) = 6700
	(d) Given a multiplication problem such as 4500 x 500 = _____, the student can name the product.	
Identity element for multiplication	(e) The student can solve equations such as 1 x 8967 = _____; 555 x _____ = 555; 89,453 - 89,453 = _____; 349 - _____ = 349; and 870 - _____ = 870 x _____.	
Multiplicative property of 0	(f) The student can solve equations such as 7320 x 0 = _____; 0 = _____ x 4900; 0 - 685 = _____; 0 x 1 = _____; and 0 - 1 = _____.	

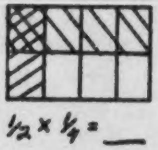
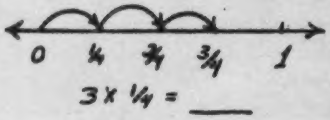
Major Topic	Behavioral Objectives	Teaching Aids
	(g) Given a division problem such as 7 - 0 = _____, the student can demonstrate that the problem has no solution by using repeated subtraction and/or the inverse relationship.	Illustration: 6/2 = 3, because 2 x 3 = 6. However, 6/0 is not valid because there is no number by which 0 may be multiplied to get 6. $\begin{array}{r} 0\overline{)\,7} \\ \underline{-0} \\ 7 \\ \underline{-0} \\ 7 \\ \underline{-0} \\ 7 \end{array}$ 7 - 0 = no number because no number x 0 = 7. etc. etc.
Distributive property of multiplication over addition	(h) The student can name the missing numbers in problems such as: $\begin{array}{rcrl} 6754 & & & \\ \underline{\times\ 342} & & & \\ \underline{} & = & 2 & \times\ 6754 \\ 270160 & = & \underline{} & \times\ 6754 \\ 2026200 & = & 300 & \times\ \underline{} \\ \hline \underline{} & = & \underline{} & \times\ 6754 \end{array}$	
5. Greatest Common Factor		
Greatest common factor (greatest common divisor)	Given a set of numbers such as 8, 12, 20, and 32, the student can name the greatest common factor of the numbers.	Review Prime Factorization from Grade 5, Section A-7-(a) and (b).

Major Topic	Behavioral Objectives	Teaching Aids
C. THE RATIONAL NUMBERS		
1. Reading and Writing the Fractional Numeral		
Rational Numbers Common fractions a/b	(a) The student can identify, name, read and write fractions for rational numbers.	Demonstrate fractional parts by using flannel board, magnetic board or chalkboard.
	(b) Given a fraction such as 3/5, the student can identify, name and distinguish the numerator and the denominator.	Example: 
Improper fractions and mixed numerals	(c) The student can rename an improper fraction as a mixed numeral and vice versa.	Example: 25/7 = 3 4/7 and 16 2/3 = 50/3
Rational Numbers	(d) The student can identify a rational number as one which can be expressed as the ratio of two whole numbers a/b when b ≠ 0.	
	(e) The student can demonstrate that all whole numbers are also rational numbers.	Example: 2 = 4/2 = 14/7 = 100/50, etc.
2. Equivalent Fractions and Inequalities		
	(a) Given a fraction such as 7/8, the student can write a set of fractions which are equivalent to 7/8.	Example: 7/8 = 14/16 = 21/24 = 28/32, etc. Transparency overlays, paper folding and fractional part kits may be used to illustrate equivalent fractions.  2/3 = 4/6

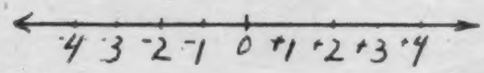
Major Topic	Behavioral Objectives	Teaching Aids
	(b) The student can rename a given fraction in simplest form.	Example: 12/32 = 3/8.
	(c) Given a mathematical sentence such as 4/9 = ?/18 = 32/? the student can name the missing numerator or denominator.	
3. Ordering Fractions	(a) Given three fractions, the student can express each as an equivalent fraction with the least common denominator.	Example: 2/3, 3/4, 1/2 are 8/12, 9/12, 6/12, respectively.
	(b) Given three fractions, the student can order them from least to greatest or greatest to least. Example: $6/12 < 8/12 < 9/12$ then $1/2 < 2/3 < 3/4$	$2/3 < 3/4$ because $8/12 < 9/12$ or $2/3 < 3/4$ because $2 \times 4 < 3 \times 3$
4. Number Pairs, Ratio and Proportion	Given a situation involving a rational number, the student can identify a/b, as a ratio, as a fraction, or as an indicated division.	Definitions: $a/b = c/d \longleftrightarrow ad = bc$ $a/b < c/d \longleftrightarrow ad < bc$ $a/b > c/d \longleftrightarrow ad > bc$
5. Decimal Fractions		
Decimal fractions -- tenths hundredths thousandths	(a) The student can identify, name, read, and write decimal numerals for rational numbers named with common fractions having denominators of 10, 100, 1000. For example: $7\ {}^{14}/_{100} = 7.14.$	

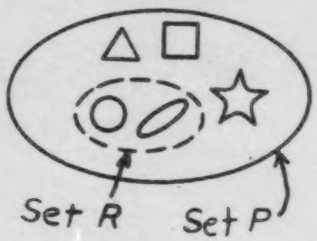
Major Topic	Behavioral Objectives	Teaching Aids
	(b) Given a numeral such as 23.74, the student can write the expanded numeral.	23.74 = (2 x 10) + (3 x 1) + (7 x 1/10) + (4 x 1/100).
	(c) Given a set of numerals such as 1/3, 2 1/2, .23, the student can classify them as common fractions, mixed numerals, or decimal fractions.	
D. THE RATIONAL NUMBERS OPERATIONS 1. Addition and Subtraction		
Definition (fractions with like and unlike denominators)	(a) Given a model (region or number line) for the addition or subtraction of rational numbers, the student can determine and name the sum or difference.	
Inverse relationship	(b) The student can solve equations such as: 3/4 + _____ = 7/8; 2 2/3 - _____ = 5/6; _____ + 3/5 = 1 2/7; _____ - 3/4 = 1 1/2	Emphasize the application of greatest common factor and least common multiple to addition and subtraction of fractions.
	(c) The student can check subtraction problems by addition.	
Algorithms Fraction notation Like denominators	(d) The student can name the sums and differences (as fractions in lowest terms and/or as mixed numerals) for problems such as: 7/11 + 3/11 = _____; 8/12 - 5/12 = _____	

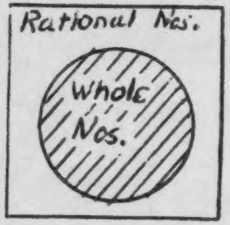
Major Topic	Behavioral Objectives	Teaching Aids
Unlike denominators renaming with common denominator)	(e) The student can name the sums and differences (as fractions in lowest terms and/or as mixed numerals) for problems such as: 3/7 + 8/9 + 13/18 = _____; 5/9 - 1/4 = _____	
Mixed numerals (renaming and regrouping)	(f) The student can name the sums and differences (as fractions in lowest terms and/or as mixed numerals) for problems such as: $\begin{array}{rcr} 7\ 1/4 & = & 7\ \ 3/12 \\ \underline{+4\ 2/3} & = & \underline{4\ \ 8/12} \\ & & 11\ \ 11/12 \end{array}$; $\begin{array}{rcrcr} 7\ 1/4 & = & 7\ 3/12 & = & 6\ 15/12 \\ \underline{-4\ 2/3} & = & \underline{4\ 8/12} & = & \underline{4\ \ 8/12} \\ & & & & 2\ \ 7/12 \end{array}$	
2. Properties of Addition and Subtraction		
	(a) The student can check subtraction problems by addition.	
Properties Commutative property of addition	(b) The student can solve equations such as: 1/2 + _____ = 1/3 + 1/2 3/8 + _____ = _____ + 3/8	
Associative property of addition	(c) The student can solve equations such as: (1/2 + 1/3) + 2/5 = 1/2 + (_____ + 2/5)	
	(d) Given an addition problem with three addends, the student can indicate the grouping of addends which will make the addition easiest by inserting parentheses.	Example: 14/33 + (21/35 + 14/35) = 1 14/33

Major Topic	Behavioral Objectives	Teaching Aids
Identity element for addition (0 = 0/1 = 0/2 = 0/3 ...)	(e) The student can solve equations such as: 1/2 + 0/8 = _____; 4/7 - _____ = 0; 0/7 + 0/10 = _____; and _____ - 0/2 = 7/15.	
3. Multiplication and Division of Fractions		
Definition of Multiplication	(a) Given a model (region or number line) for the multiplication of two rational numbers, the student can determine and name the product.	 
Definition of division (inverse relationship)	(b) Given a division equation such as 5/12 - 1/4 = _____, the student can write and solve the related multiplication equation, 1/4 x _____ = 5/12.	Definition: a - b = c because b • c = a. Example: 1/3 - 1/6 = 2 because 1/6 x 2 = 1/3
Algorithms Fraction notation Multiplication	(c) The student can name the products (as fractions in lowest terms) for problems such as 3/4 x 2/5 = _____.	

Major Topic	Behavioral Objectives	Teaching Aids
4. **Properties of Multiplication and Division**		
Commutative property of multiplication	(a) The student can check division problems by multiplication.	
	(b) The student can solve equations such as: $1/2 \times \_\_\_\_ = 1/3 \times 1/2$ $3/8 \times \_\_\_\_ = \_\_\_\_ \times 3/8$	
	(c) The student can solve equations such as: $(1/2 \times 1/3) \times 2/5 = 1/2 \times (\_\_\_\_ \times 2/5)$	
Identity of Multiplication	(d) The student can solve equations such as: $5/7 \times 3/3 = \_\_\_\_$	
3. **Operations With Decimals**		
Decimal notation (including money)	(a) The student can name the sums and differences for problems such as: $5.26 + 39.1 + .832 = \_\_\_\_$ $78.02 - 13.751 = \_\_\_\_$	
Decimal notation (money)	(b) The student can name the products and quotients for problems such as: $\begin{array}{r} \$\ 2.53 \\ \times\ 27 \\ \hline \end{array}$ $\quad 45\overline{)\$41.85}$	

Major Topic	Behavioral Objectives	Teaching Aids
E. THE INTEGERS		
1. The Negative Rationals	The student can name the numbers associated with points on the number line in both the positive and negative direction from zero.	Have a large number line on display in the room.  Emphasive the two characteristics of distance and direction.
2. Addition and Subtraction of Integers	Given a set of integers having less than three digits, the student must be able to determine the sum and the difference of any two.	Refer to the Postman Stories in the addenda.
3. Multiplication and Division of Integers	(a) Given a set of integers having less than three digits, the student must be able to determine the product of any two. (b) Given a set of integers having less than three digits, the student must be able to determine the quotient of any two that are evenly divisible.	Refer to the Postman Stories in the addenda.

Major Topic		Behavioral Objectives	Teaching Aids
A. SETS			
1. Collections		Given a description of a set, the student can distinguish between members of the set and things which are not members.	For example: Given the set of letters in the alphabet, the student can select the set containing the letters in the word "arithmetic".
2. One-to-One Correspondence		Given two equivalent sets, the student can demonstrate a one-to-one matching between the members of the sets.	See the previous grades for explanation and pictures for matching sets.
3. One-to-Many Correspondence		The student can describe familiar physical situations in which a one-to-many matching occurs.	For example: One dog has four legs.
4. Equivalent and Non-equivalent Sets	(a)	Given two sets, the student can identify them as equivalent or non-equivalent.	For example: The set of students and the set of chairs in the room.
Empty Set	(b)	The student can identify and describe sets which have no members.	See previous grades for examples.
Subsets	(c)	Given a set, the student can identify and describe a subset.	 Set R is a subset of Set P which means Set R is contained in Set P, and is written as $R \subset P$

Major Topic		Behavioral Objectives	Teaching Aids
5. Cardinal Number 0 - 1,000,000,000		The student can determine the cardinal number of a given set.	
6. Commutative, Associative, and Distributive, Property		The student shall be able to demonstrate the commutative, associative, and distributive properties through the use of sets.	See Grade Three, Section A-7(a), (b), (c)
7. Equal Sets		Given two sets, the student can identify them as equal or not equal.	Set A = $\{a, b, c\}$, Set B = $\{c, b, a\}$, Set C = $\{a, e, d\}$, $A = B$, $A \neq C$
8. Union and Intersection of Sets		Given two sets (disjoint or not disjoint), the student can give the set that is the union and the set that is the intersection of the two sets.	See Grade Four, Section A-10(c)
9. Finite Infinite Empty		The student can classify a given set as: Finite---has a definite number of members. Infinite---has an unlimited number of members. Empty---has no members.	
10. Venn Diagrams		The student can construct a Venn diagram to show the relationship of whole numbers to rational numbers.	
11. Set Symbolism	(a)	The student shall be able to show set union and intersection using set symbolism.	$A = \{1, 2, 3\}$ $B = \{4, 5, 6\}$ $A \cup B = \{1, 2, 3, 4, 5, 6\}$ $C = \{1, 2, 3, 4\}$ $D = \{2, 4\}$ $C \cap D = \{2, 4\}$
	(b)	The student can show that one set is a subset of another using set symbolism.	$A \subset C$ $D \subset C$ $B \not\subset A$
	(c)	The student can give the number of a set using set symbolism.	$n(A) = 3$ $n(A \cup B) = 6$ $n(C \cap D) = 2$

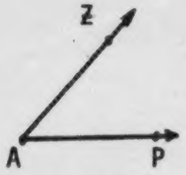
Major Topic	Behavioral Objectives	Teaching Aids
B. RELATIONS AND FUNCTIONS		
1. "What's My Rule?"	Given the game situation described in the addenda, the student will be able to determine the missing values, and state the rule of the game.	See In/Out tables below.
2. Number Patterns	The student shall be able to complete the next three elements of the following sets. A = {3, 6, 9, ___, ___, ___, ...} B = {4, 10, 16, ___, ___, ___, ...}	Have students form number patterns. See addenda.
C. MATHEMATICAL SENTENCES		
1. Open, True and False Sentences	(a) The student shall be able to classify the following types of sentences as open, true, or false, and answer or correct the open and false sentences. $2 + \square = 3$; $4 - 3 = 2$; $4 \times 3 = 12$; $14 \div 2 = \square$	

In	Out	In	Out	In	Out
1	4	0	0	(1,2)	2
2	6	1	3	(2,3)	6
3	8	2	6	(3,4)	12
4	?	3	?	(4,5)	?
5	?	4	?	(6,7)	?

Major Topic	Behavioral Objectives	Teaching Aids
≠ > <	(b) The student shall be able to use the "is not equal," "less than" and "greater than" signs in mathematical sentences.	9 ≠ 10; 7 3 + 5; 8 4 - 1
	(c) The student shall be able to use parentheses to show order in a mathematical sentence involving addition, subtraction, multiplication, and division.	(7 - 4) + (2 x 5) - (6 - 2) = □
	(d) The student can find the solution set to mathematical sentences with three variables. For example: 3 x 321 = (3 x □) + (3 x △ + (3 x ⬡)	Where different shapes are used for quantities, the numbers entered may be different or the same; where the shapes are the same, the numbers entered must be the same.
	(e) The student can find the solution to problems where placeholders are used in the numerators of fractions.	$\frac{\square}{5} + \frac{\triangle}{5} = \frac{4}{5}$
	(f) The student can translate an English sentence into a mathematical sentence and solve. For example: A boys' club with eleven members bought a flag for \$5.61. If they shared the cost equally, how much did each boy pay?	\$5.61 - 11 = □

GEOMETRY

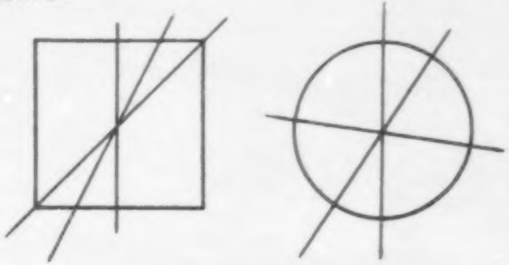
GRADE FIVE

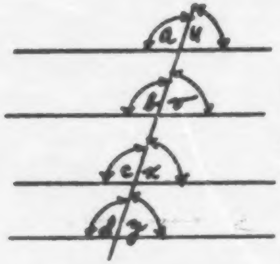
Major Topic	Behavioral Objectives	Teaching Aids
A. RECOGNITION AND DESCRIPTION OF PLANE FIGURES (As Sets of Points)		
1. Plane Figure Point Path (curve) Line AB Line Segment AB	(a) The pupil can describe a plane geometric figure as a set of points.	For example: A circle is the *set of all points* in a plane a fixed distance from a given point.  *Angle* Z A P
Ray AB Angle (vertex) Right angle	(b) The student can demonstrate that an angle is the *set of all points* contained in the union of two rays with a common endpoint.	*Written* $\angle ZAP$
Quadrilateral	(c) The student can read and write standard notation for the plane figures named on the left.	 *Line Segment* HO *Written* $\overline{HO}$
Parallelogram Square Rectangle Rhombus	(d) Given models of the plane geometric figures named on the left (wire, paper or flannel cutouts, pencil or chalk outlines, etc.), the student can *identify, name,* and *distinguish* among them.	
Pentagon Hexagon Octagon		
Circle Center Radius Diameter Chord Circumference		Students may be led to discover the circumference-diameter relation by measuring these, but simple diameters such as "1", "2", "3", "4" should be used first so as to not obscure the result.

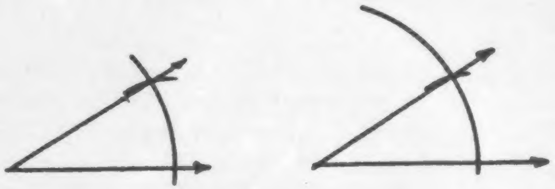
Major Topic	Behavioral Objectives	Teaching Aids
2. Space Figures Right Rectangular Prism Cube Sphere Cylinder Cone Pyramid	(a) Given models of the space figures named on the left (wood or plactic solids, paper models, sketches, etc.), the child can *identify*, *name*, and *distinguish* among them.	A right rectangular prism may be used to help develop the notion of three planes can determine a point. Each surface may be given a name. These surfaces may be thought of as resulting in a single unique point (corner or vertex).
	(b) The student will distinguish between models of common polyhedra, such as a tetrahedron, a cube, or a rectangular prism.	
	(c) The student can *point to* and thus *identify* faces, edges, vertices, and diagonals of common polyhedra.	
B. RELATIONS OF GEOMETRIC FIGURES		
1. Determining a Plane	A student can show that a plane is determined by three points not all on one line.	The student using paper or chalkboard should explain why two points cannot determine a plane, and that three non-collinear points can determine a plane and may be called *coplanar points*.

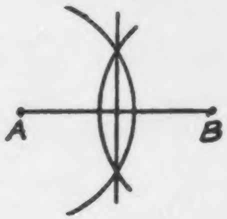
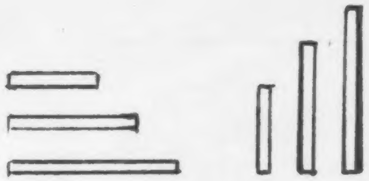
GEOMETRY GRADE FIVE

Major Topic	Behavioral Objectives	Teaching Aids
2. Similarity	(a) The student will choose those pairs of shapes that are *similar*.	Certain classes of shapes are always similar (squares, circles, equilateral triangles to name a few). Tests of similarity should not become too involved in the elementary grades. Rectangles are frequently not similar. It may suffice at this level to have pupils recognize that enlarging a shape to a similar one, multiplies all the dimensions by a common factor (or divides all the dimensions if it is to be made smaller). Shapes such as rectangles then should be easily perceived as having dimensions which are multiples of a model. Triangles, whose angles fit each other and whose sides are double or triple those corresponding sides of a model, can be used.
	(b) The student will demonstrate the similarity of maps made with different scales.	Using a unit of measure from either two maps with different scales, the student should verify that a proportionate relation exists, that for a single unit on one map, a corresponding increase (as twice the unit) results in a corresponding value on the second map.

Major Topic	Behavioral Objectives	Teaching Aids
3. Symmetry	The student will demonstrate *understanding of symmetry* with respect to a point by folding a paper along a line through the center of such geometric figures as a square. 	Child can be asked to distinguish whether figures have axial or line symmetry (reflecting the object about a line in the plane) or rotational symmetry (the shape will fit again after being turned round in the plane less than a complete revolution). A figure is said to have *point symmetry* if all line segments that can be drawn through that point and terminated by the figure are bisected by that point.
4. Congruency	(a) Given a pair of line segments, angles, triangles, or other polygons, the student can *identify the pairs* as congruent or *not* congruent by matching the figures in some manner (trace and overlay, cutouts, etc.). (b) Given several triangle models or tracings, some of which are congruent, the student will select those which are congruent.	
C. CONSTRUCTION - APPROXIMATE & EXACT		
1. Properties		
Parallel lines	(a) The student can *sketch, describe,* and *give* examples of parallel lines.	For example: Parallel lines are lines in the same plane which never intersect. The opposite edges of the table are parallel.

Major Topic	Behavioral Objectives	Teaching Aids
1. **Properties** Parallel lines (continued)		Children may be asked what name can be given to the lines on note paper. If these lines are cut by a transversal (as by a line drawn with a ruler), children may be asked to check the equivalence of corresponding angles.  *Angles a,b,c,d, are corresponding and equal; so are angles u,v,x,y.* Once there is an intuitive acceptance of this, students may be asked to find those lines on a page which are parallel and label them.

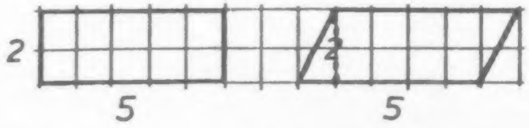
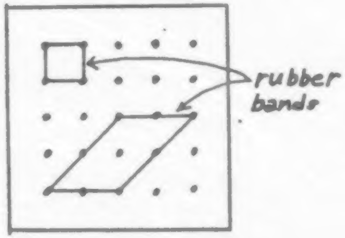
Major Topic	Behavioral Objectives	Teaching Aids
1. Properties (continued)		
Intersecting Lines	(b) The student can *sketch, describe*, and *give* examples of intersecting lines.	For example: Intersecting lines are lines that have one point in common. Maryland Parkway intersects Desert Inn Road. The cross-hairs on my rifle sight intersect.
Perpendicular Lines	(c) The student can *sketch, describe*, and *give* examples of perpendicular lines.	For example: Perpendicular lines are intersecting lines that form right angles. The top edge and side edge of the window are perpendicular to each other. A model of a right angle may be used to test for this relation.
Plane Geometric Figures	(d) Using a straightedge, the student can construct models for lines, line segments, rays, angles, and polygons and label them.	
Copy	(e) Using a straightedge and compass, the student can construct a plane figure *congruent* to a given line segment, angle, or triangle.	For example: 

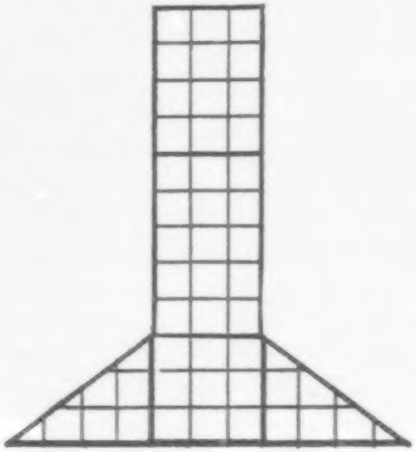
Major Topic	Behavioral Objectives	Teaching Aids
Line Segment Bisector Angle Bisector	(f) Using a straightedge and compass, the student can bisect (separate into two congruent figures) a given line segment and given angle.	For example: 
	(g) Using a compass, the student can construct a circle with a given center and radius (or diameter).	
	(h) Demonstrate an understanding of various polyhedra by making appropriate paper models.	Refer to *LEARN TO FOLD-FOLD TO LEARN* by Janet Abbott, Franklin Publications, Inc., Pasadena, California.
D. COORDINATE SYSTEMS AND GRAPHS		
1. Graphs	(a) Given suitable information, the student will construct simple picture, bar and line graphs.	For example:  Two ways to represent same data.

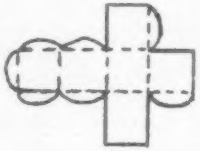
Major Topic	Behavioral Objectives	Teaching Aids
1. Graphs (continued)	(b) The student can use the number line to represent positive rational integers.	
	(c) Use the number line to represent negative integers.	
E. CIRCUMFERENCE		
Plane Figures	Given several cutout circles or cans with even diameters, the pupil will measure both the circumferences and diameters and attempt to discern the consistent relationships between them.	The teacher can take a set of circles with varying diameters, have the children record the diameters. If the teacher takes a circle with a 4-cm diameter and then measures the circumference (that is, has the students measure the circumference), they should find it to be about 12.56 cm. Then have the children find the ratio of the circumference to the diameter .... about 3.14. Now the teacher asks the children "If we take the circle with a greater diameter, say six cm, can we expect the diameter to be greater or less? Or might it be about the same? Let's find out." So they find the circumference to be about 18.84 cm. The ratio is again 3.14.

MEASUREMENT AND ESTIMATION GRADE FIVE

Major Topic		Behavioral Objectives	Teaching Aids
A. CONCEPT OF MEASUREMENT			
1. Process of Measuring	(a)	Given a measurable physical property such as length, area, weight, temperature, etc., the student can select a suitable unit and/or measuring device and measure the property. For example: Weight can be measured by using a _____. Time can be measured by using a _______.	Tell story of some of our early systems of measure. Let the students select their own and then decide which is the best for measuring objects. (Length and width of room, weight (mass) of book, weight of a rock, etc.)
Arbitrary Selection of Unit	(b)	The student can describe the relationship between the weight (mass) of water at 4°C. and volume of that water.	Use laboratory material. See Addenda.
Approximate Nature of Measurement	(c)	The student can demonstrate his understanding of the approximate nature of measurement by stating the precision of the measure. For example: The volume of a bottle is 14 ml correct to the nearest ml.	Use laboratory material. See Addenda.
2. Concept of Graphs		The student can obtain information from line graphs to be used in computational problems.	Make a collection of different types of graphs. Have students bring graphs from newspapers and magazines. Use road maps.

Major Topic	Behavioral Objectives	Teaching Aids
B. MEASUREMENT OF PHYSICAL PROPERTIES		
1. Time Second, minute, hour, year, decade, century. Time zones.	(a) Given that the time is 9:00 a.m. in Boise, the student can name the time in Spokane, Chicago, and New York. (b) The student can name the century for a given date. For example: 1492 was in the 15th century.	Use actual clocks for telling time. Have time activities, such as: How long does it take to get ready for recess? Discuss history of time. Show maps with time zones marked on them.
2. Money	The student can do addition, subtraction, multiplication, and division problems that involve money.	Have a shopping trip; pick a lunch. See addenda. Adapt games to grade level.
3. Linear Measurement Length Millimeter, centimeter, meter, kilometer	(a) The student can use various measuring devices (ruler, meter stick) to measure length in whole and fractional parts of units.	Provide a list of objects. Have the class measure these objects. Record the data and compare results. What conclusions can be drawn?
Perimeter	(b) Given the measures of the sides of a polygon, the student can compute the perimeter.	Have large cardboard objects (rectangles, triangles, polygons) for the children to measure.
4. Area Square units related to units of length Rectangle Parallelogram	(a) Using paper figures and scissors, the student can demonstrate that the area of a parallelogram and a rectangle having the same base and altitude measures are equal. For example: 	Use geo-board with clear plastic overlays and checkerboard grid. 

Major Topic	Behavioral Objectives	Teaching Aids
4. Area (continued)	(b) The student will use graph paper to assemble models of cubes, prisms, etc. to determine surface area.	Have the students build cubes, rectangular solids, prisms, etc. out of squared graph paper to help them develop concepts about measure of surface area. Students should note that as they make a pattern for their models, they can predict easily what the surface will be, simply by counting squares. They might also be led to note that the volume of the completed solid is not as easily determined by merely looking at the pattern. For example, the pattern for a triangular-based prism follows:  Leave enough paper as tabs around the pattern to glue it together.

Major Topic	Behavioral Objectives	Teaching Aids
4. Area (continued)	(c) Given the measures of the base and altitude of a rectangle or parallelogram, have the student compute the area.	
5. Volume Cubic units related to units of length	(a) Given a sketch of a right rectangular space figure, the student can determine and name the volume of the figure by counting the number of volume units (cubes) needed to fill the space.	Use several cubic centimeter blocks either constructed out of paper or wood and several larger boxes of different sizes and shapes to determine volume. Pattern for a cube 
Liquid Measure 250 ml, 500 ml	(b) The student can name the standard metric units of liquid measure and measure the capacity of a given container to the nearest whole unit.	Use laboratory material and a set of questions pertaining to liquid measure. How many 250 ml containers does it take to fill a large milk carton?
6. Weight Gram, kilogram, metric ton	Using a scale, the student can measure the weight of a given object in whole and fractional parts of units.	Have a collection of various objects to be weighed. Have the children lift the objects and then place them in rows according to their relative guesses as to weight. Put their guesses on the board. Weigh one of the objects, then allow the children to reevaluate their estimates. Weigh objects and compare with the estimates.
7. Temperature Celsius degrees	Using a thermometer calibrated in Celsius degrees, the student can read the temperature to the nearest degree and name the freezing point and boiling point of water.	Use demonstration thermometer. Using a candy thermometer, determine the boiling point of water. Try to obtain a variety of thermometers for the children to read. (Meat thermometer, clinical thermometer, candy thermometer, indoor-outdoor.)

Major Topic	Behavioral Objectives	Teaching Aids
8. Metric System	The student can name the metric measures, millimeter, centimeter, meter, liter and gram, and tell objects that would be measured by each.	Get a meter stick, set of metric weights; then use a laboratory set of items and let the student decide what objects can be measured.
9. Angle Degree	Using a protractor the student can measure an angle to the nearest degree.	Use large demonstration model of a protractor, either of wood or cardboard.
10. Graphs	(a) The student can make picture, bar, and line graphs for data that has been collected.	Have students collect and organize data using bar, line or picture graphs. You might use problems involving the class, such as: Range of weights, boys and girls Range of heights, boys and girls Color of hair Color of eyes Hobbies
Statistics	(b) The student can find the mode, median, and mean from experimental data.	From the above data collected, the student can determine mode, median and mean.
	(c) Given a set of numbers such as 98, 75, 83, 100, and 79, the student can name the average (arithmetic mean) of the numbers.	
C. RENAMING MEASURES		
1. Comparison of Units and Conversion of Units	The student can express the relationships between units of measure appropriate to the grade level and can rename a measure in other units. For example: 248 hours = 10 days, 8 hours = 1 week, 3 days, 8 hours	

Major Topic	Behavioral Objectives	Teaching Aids
D. COMPUTATION WITH MEASURES 1. Addition, Subtraction, Multiplication, Division	The student can compute with measures appropriate to the grade level, assign the proper unit to the result, and rename if necessary. For example: $\begin{array}{l} 1.25 \text{ meters} \\ \underline{\text{X}\ \ 6\qquad\quad} \\ 7.50 \text{ meters} \end{array}$ $\begin{array}{l} \ \ 2 \text{ hrs. } 13 \text{ min.} \\ \underline{\ \ \text{X}\ \ 6\qquad\qquad} \\ 12 \text{ hrs. } 78 \text{ min.} = 13 \text{ hrs. } 18 \text{ min.} \end{array}$	

NUMBER AND OPERATIONS

GRADE SIX

Major Topic	Behavioral Objectives	Teaching Aids
A. THE NATURAL NUMBERS COUNTING AND NUMERATION		
1. Counting the Number in a Set.	Given a list of *finite* and *infinite* sets, the student can identify those which are *finite* (the set of students in the sixth grade) and those which are *infinite* (the set of counting numbers).	
2. Reading, Recognizing, and Writing Numbers.		
Names for Numbers	(a) The student can identify, name, read, and write many different names for the same number. For example: $8 = 8/1 = 32/4 = 8.00 = 2^3 = \text{VIII} = 13_{\text{five}}$ etc.	
Whole Numbers 0 - Infinity	(b) The student can identify, name, read, and write numerals for whole numbers.	
	(c) Given a fifteen-digit numeral, the student can read it and write it in words.	Display a chart indicating place value and period value.
	(d) Given a verbal phrase naming a number, the student can write the number words and/or the Arabic numeral.	

10^8	10^7	10^6	10^5	10^4	10^3	10^2	10^1	10^0
millions			thousands			ones		
3	7	8	9	5	2	8	0	1

Major Topic	Behavioral Objectives	Teaching Aids
3. Using Ordinal Names		
Order Relations	(a) Given a set of whole numbers, the student can write them in order from least to greatest and vice versa.	
	(b) The student can write the symbols , , =, ≠ to express the relationship between numbers.	
4. Place Value		
Place Value -- Fifteen Digit	(a) Given a base ten (decimal) numeral, the student can identify and name the *period* value for each group of three digits and the *place* value for each digit.	
Expanded Notation	(b) Given a numeral such as 473,245, the student can write the expanded numeral: $473{,}245 = (4 \times 100{,}000) + (7 \times 10{,}000) + (3 \times 1000) + (2 \times 100) + (4 \times 10) + (5 \times 1)$.	Refer to chart with Grade Six, Section A-2-(c) under "Teaching Aids."
	(c) Given a numeral such as 81,437, the student can write the expanded numeral using *exponential* notation. $81{,}437 = (8 \times 10^4) + (1 \times 10^3) + (4 \times 10^2) + (3 \times 10^1) + (7 \times 10^0)$.	Review exponents Example: $10^3 = 10 \times 10 \times 10$

Major Topic	Behavioral Objectives	Teaching Aids
5. **Non-decimal Numeration**		
Roman Numerals through M	(a) Given a numeral such as 1,151, the student can write the Roman numeral MCLI.	Display a chart showing specific values of Roman numerals.
	(b) Given a Roman numeral such as MCCXLV, the student can write the Arabic numeral 1,245.	Have the students find examples of applications of Roman numeral use.
	(c) Given the numerals XXX and 333, the student can describe the difference between them by saying: In the Roman numeral, the "X" always has the value of ten, but in the decimal numeral, the value of the "3" depends upon its *place* in the numeral. Therefore, XXX = 10 + 10 + 10 but 333 = 300 + 30 + 3.	
	(d) Given the numeral CDLX, the student can write the numeral (500 - 100) + 50 + 10.	
Non-Decimal Numeration -- Base two, four, five	(e) Given a set with from 1 to 100 members, the student can group the members and write a base five numeral for the cardinal number of the set.	Have the students construct place value charts for base five. Use grouping of tongue depressors or other aids.

$5^4 = 625$	$5^3 = 125$	$5^2 = 25$	5	units
2	3	3	4	1_{five}

Major Topic	Behavioral Objectives	Teaching Aids
5. Non-decimal Numeration (continued)	(f) Given a numeral such as 10110_{two}, the student can write the expanded numeral: $10110_{two} = (1 \times 16) + (0 \times 8) + (1 \times 4) + (1 \times 2) + (0 \times 1)$. $10110_{two} = (1 \times 2^4) + (0 \times 2^3) + (1 \times 2^2) + (1 \times 2^1) + (0 \times 2^0)$.	
6. Rounding and Estimation	Given a numeral such as 3,628.765, the student can round it to the nearest hundredth, tenth, one, ten, hundred, and thousand.	Use a number line to illustrate *closeness* for rounding.
7. Primes and Composites	(a) Given a set of whole numbers, the student can classify them as *prime* or *composite*.	Refer to Grade V, Section A-7-(a) and (b).
	(b) The student can express a composite number as the product of prime numbers. For example: $28 = 2 \times 2 \times 7$	
8. Exponential Notation		
Exponentiation	(a) Given a problem such as: $5^3 = \_\_\_\_\_$, the student can identify and name the base, exponent and power.	$5^3 = \underline{125}$ 5 is the base. 3 is the exponent. 125 is the third power of 5.
Exponential Notation	(b) The student can write expanded numerals using exponential notation.	

Major Topic	Behavioral Objectives	Teaching Aids
8. Exponential Notation (continued)	(c) Given a numeral such as 3^5, the student can rename it as (3 x 3 x 3 x 3 x 3) or 243.	
	(d) Given the numerals 2^3 and (2 x 3), the student can distinguish between them and and that they do *not* name the same number!	$2^3 = 8$; $2 \times 3 = 6$
Scientific Notation	(e) The student can rename a given numeral using scientific notation.	$93{,}000{,}000 = 9.3 \times 10^7$ and $34{,}589 = 3.4589 \times 10^4$.
B. THE NATURAL NUMBERS OPERATIONS		
1. Addition and Subtraction		
Whole Numbers	(Review and maintain concepts and skills).	
Inverse relationship	(a) The student can solve addition or subtraction problems with missing sums, differences, or addends.	_____ + 45,987 = 6,000,001 and _____ - 500,800 = 1,333,708
	(b) The student can check subtraction problems by addition.	
Basic Facts Through Sums of 18	(c) Given any single-digit addition or subtraction combination, the student can **immediately* name the sum or difference.	

**Immediately* is defined as 5 seconds or less.

Major Topic	Behavioral Objectives	Teaching Aids
B. THE NATURAL NUMBERS OPERATIONS (continued)		
Algorithms Column Addition and Subtraction	(d) Given any "reasonable" addition or subtraction problem, the student can name the sum or difference.	
Other Notation	(e) The student can identify and name sums, differences, missing addends, missing digits, and missing operational signs in problems written in both horizontal and vertical notation.	
2. Properties of the Natural Numbers		
Properties Commutative and Associative Properties of Addition	(a) Given an addition problem with three or more addends, the student can demonstrate how to find the sum in the easiest way of renaming and rearranging the addends.	3 + 5 + 7 = 3 + (5 + 7) = 3 + (7 + 5) = (3 + 7) + 5 =
Identity Element for Addition	(b) The student can solve equations such as: 3905 + _____ = 3905; 5477 - _____ = 5477 + _____; 0 = _____ - 380,357; and 8,777,300 = _____ - 0.	

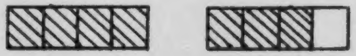
Major Topic	Behavioral Objectives	Teaching Aids
3. Multiplication and Division	(Review and maintain concepts and skills).	
Inverse Relationship	(a) The student can solve multiplication or division problems with missing products, quotients, or factors.	_____ x 317 = 1,595,144 and _____ ÷ 23 = 11,776
	(b) The student can check division problems by multiplication (without remainder) or by multiplication and addition (with remainder).	
Basic Facts	(c) Given any single-digit multiplication or division combination, the student can **immediately* name the product or quotient.	Refer: addenda for activities, records and tapes. See Grade V, Section B-3-(d) under "Teaching Aids".
Algorithms Multiplication Vertical Notation	(d) Given any "reasonable" multiplication problem, the student can name the product.	
Long Division	(e) Given any "reasonable" division problem, the student can name the quotient in either of two ways.	Example: 29 R 11 and 29 11/23 23) 678 and 23) 678 46 46 218 218 207 207 11 11 Check: (23 x 29) + 11 = 678 and 23 x (29 - 11/23 = 678

**Immediately* is defined as 5 seconds or less.

Major Topic	Behavioral Objectives	Teaching Aids
3. Multiplication and Division (continued)		
Short Division	(f) Given any "reasonable" division problem with a single-digit divisor, the student can use the short algorithm (short division form) to name the quotient and remainder.	
Other Notation	(g) The student can identify and name products, quotients, missing factors, missing digits, and missing operational signs in problems written in both horizontal and vertical notation.	
4. Properties of Natural Numbers		
Properties - Commutative and Associative Properties of Multiplication	(a) Given a multiplication problem with two or more factors, the student can demonstrate how to find the products in the easiest way by rearranging the factors.	4 x 359 x 25 = (4 x 25) x 359 = 100 x 359 = 35,900
	(b) The student can check multiplication problems by reversing the order of the factors and multiplying again.	
Identity Element for Multiplication	(c) The student can solve equations such as:	1 x 8967 = ____; 89,453 ÷ 89,453 = ____; 349 ÷ ____ = 349; and 870 ÷ ____ = 870 x ____.
Multiplicative Property of 0	(d) The student can solve equations such as:	7320 x 0 = ____; 0 = ____ x 4900; 0 ÷ 685 = ____; 0 x 1 = ____; and 0 ÷ 1 = ____.

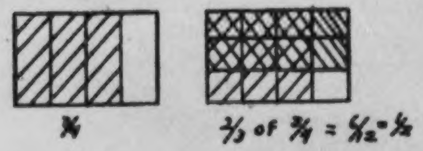
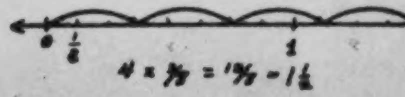
Major Topic	Behavioral Objectives	Teaching Aids
4. Properties of Natural Numbers (continued)		
Multiplicative Property of 0	(e) Give a division problem such as: 15 - 0 = _____, the student can demonstrate that the problem has no solution by using *repeated subtraction* and/or the *inverse relationship*.	*REPEATED SUBTRACTION* 0) 15 - 0 1 15 - 0 3 15 - 0 25 15 etc. etc. etc. *INVERSE RELATIONSHIP* 15 ÷ 0 = no number because no number x 0 = 15
Distributive Property of Multiplication over Addition	(f) Given a problem such as: 53 x 624 = _____, the student can demonstrate his understanding of the distributive principle by multiplying in expanded horizontal form.	53 x 624 = (50 + 3) x 624 = (50 x 624) + (3 x 624) = 31,200 + 1872 = 33,072

Major Topic	Behavioral Objectives	Teaching Aids
5. Greatest Common Factor-Least Common Multiple		
Greatest Common Factor (greatest common divisor)	(a) Given a set of numbers such as: 28, 56, 70, and 126, the student can name the greatest common factor of the numbers.	Review Prime Factorization Refer: Grade V, Section A-1 (a) and (b).
Least Common Multiple (least common denominator)	(b) Given a set of numbers such as: 12, 15, 35, and 36, the student can name the least common multiple of the numbers.	
C. THE RATIONAL NUMBERS		
1. Reading and Writing the Fractional Number		
Common Fractions a/b	(a) Given a model of five-sevenths, the student can identify, name, read, and write a numeral (fraction) for the rational number associated with the model.	Refer: Grade V, Section C-1 (a), "Teaching Aids".
	(b) Given a fraction such as: 7/3, the student can identify, name and distinguish the *numerator* and the *denominator*.	
Equivalent Fractions	(c) Given a fraction such as: 7/8, the student can write a set of fractions which are equivalent to 7/8.	$\left\{ \frac{7}{8}, \frac{14}{16}, \frac{21}{24}, \frac{28}{32}, \ldots \right\}$

Major Topic	Behavioral Objectives	Teaching Aids
Improper Fractions	(d) Given a model such as:  the student can identify, name, read, and write the fraction 7/4 and/or the mixed numeral 1 3/4 for the rational number associated with the model.	Demonstrate fractional parts by using magnetic board, flannel board, or chalkboard.
	(e) The student can rename an improper fraction as a mixed numeral and vice versa.	25/7 = 3 4/7 and 16 2/3 = 50/3
2. Equivalent Fractions and Inequalities	(a) The student can rename a given fraction in simplest form.	12/32 = 3/8.
	(b) Given a mathematical sentence such as 4/9 = ?/18 = 32/?, the student can name the missing numerator or denominator.	Refer: Grade V, Section A-2 (a).
	(c) Given a set of fractions such as: 3/4, 3/5, 6/8, 18/24, 18/30, 6/10, 75/100, 15/25, 60/100, the student can identify and name the fractions which are equivalent.	9/12 = 6/8 Because 9 x 8 = 12 x 6 or Because 9/12 = 18/24 and 6/8 = 18/24
3. Ordering Fractions		
Order Relations	Given a set of rational numbers such as: 1/8, .33..., 2/3, 1/4, .5, 7/4, 5/6, .75, and 4/3, the student can arrange them in order from least to greatest and vice versa.	Refer: Grade V, Section C-3 (b)

Major Topic	Behavioral Objectives	Teaching Aids
4. Number, Pairs, Ratio, and Proportion		
Ratio	(a) The student can write the ratio of three apples to four oranges as 3:4 and/or 3/4, and given the symbol 4/9, the student can express it in words as "the ratio of 4 to 9."	
Indicated Division	(b) Given a numeral such as: 25/4, the student demonstrates his understanding of the symbol as an indicated division by renaming it as 6 1/4 and 6.25.	
Percent Notation	(c) The student can rename numerals such as: 47% as .47 and/or 47/100.	
	(d) Given a set of numerals such as 3/4, 16.4, 3 2/3, 26%, the student can classify them as common fractions, decimal fractions, mixed numerals, or percents.	
	(e) The student can name the products, quotients and missing factors for problems involving percents.	Examples: Using proportions $25\% = \frac{25}{100}$ 25% of 200 is what number? $\frac{X}{200} = \frac{25}{100}$ 50 is 25% of what number? $\frac{50}{X} = \frac{25}{100}$ 50 is what percent of 200? $\frac{50}{200} = \frac{X}{100}$

Major Topic	Behavioral Objectives	Teaching Aids
Percent Notation	(e) (continued)	Examples (continued): or Using number sentences 25% of 160 is what number? $\frac{25}{100} \times 160 = \square$ 50% of what number is 346 $\frac{50}{100} \times \square = 346$ What % of 56 is 7? $\frac{\square}{100} \times 56 = 7$
5. Decimal Fractions Tenths Hundredths Thousandths Ten Thousandths Hundred Thousandths	(a) The student can identify, name, read, and write decimal numerals for rational numbers named with common fractions having denominators of 10, 100, 1000.	7 14/100 = 7.14
	(b) Given a numeral such as: 23.74, the student can write the expanded numeral.	23.74 = (2 x 10) + (3 x 1) + (7 x 1/10) + (4 x 1/100).
	(c) Given a numeral such as: 2,479.6305, the student can read it and write it in words.	
	(d) Given a numeral such as: .17856, the student can write the expanded numeral.	.17856 = 1/10 + 7/100 + 8/1000 + 5/10,000 + 6/100,000.
	(e) Given numerals such as: .33... and .5, the student can distinguish between repeating and terminating decimal fractions.	NOTE: .33... ≠ .33

Major Topic		Behavioral Objectives	Teaching Aids
1. **Addition and Subtraction of Fractions** (continued)			
Mixed Numerals	(d)	The student can name the sums and differences (as fractions in lowest terms and/or as mixed numerals) for problems such as: $\begin{array}{rcr} 7\ 1/4 & = & 7\ \ 3/12 \\ \underline{+4\ 2/3} & = & \underline{4\ \ 8/12} \\ & & 11\ 11/12 \end{array}$ $\begin{array}{rcrcr} 7\ 1/4 & = & 7\ 3/12 & = & 6\ 15/12 \\ \underline{4\ 2/3} & = & \underline{-4\ 2/3} & = & \underline{4\ \ 8/12} \\ & & & & 2\ \ 7/12 \end{array}$	
2. **Properties of Addition and Subtraction**			
Commutative and Associative Properties of Addition	(a)	Given an addition problem with three or more addends, the student can demonstrate how to find the sum in the easiest way by renaming and rearranging the addends.	7/8 + 2/3 + 5/8 + 1 5/6 = (7/8 + 5/8 + 4/6 + 1 5/6) = 1 1/2 + 1 + 1 1/2 = 4
	(b)	The student can check subtraction problems by addition.	
Identity Element for Addition (0 = 0/1 = 0/3 ...)	(c)	The student can solve equations such as: 1/2 + 0/8 = _____; 4/7 - _____ = 0; 0/7 + 0/10 = _____; and _____ - 0/2 = 7/15.	
3. **Multiplication and Division of Fractions**			
Definition of Multiplication	(a)	Given multiplication problems such as: 2/3 x 3/4 = _____ and 4 x 3/8 = _____, the student can demonstrate how to find the products by using regions or a number line.	 

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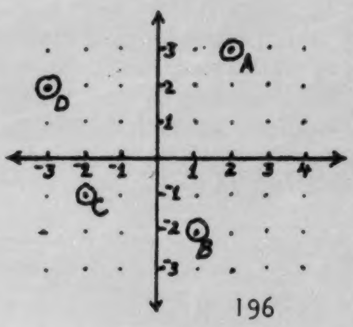
Major Topic	Behavioral Objectives	Teaching Aids
Inverse Relationship	(d) The student can solve equations such as: 1/3 x _____ = 2/15 2/7 - _____ = 3/4 _____ x 3/8 = 5/9 _____ - 1/5 = 4/11	
Algorithms Fraction Notation Multiplication	(e) The student can name the products (as fractions in lowest terms) for problems such as: 14/15 x 12/35 = _____ and 7/5 x 15/18 x 10/21 = _____ and use the reducing shortcut where appropriate.	(3 the common factor) $\frac{\overset{2}{\cancel{14}}}{\underset{5}{\cancel{15}}} \times \frac{\overset{4}{\cancel{12}}}{\underset{5}{\cancel{35}}} = \frac{8}{25}$ (7 the common factor)
	(f) The student can name the products (as fractions in lowest terms and/or as mixed numerals) for problems such as: 3 1/2 x 4/7 = _____ and 2/3 x 5 1/8 = _____ by renaming as improper fractions and/or by applying the distributive principle.	
	(g) The student can name the quotients (as fractions in lowest terms and/or as mixed numerals) for problems such as: 3/5 - 7/8 = _____.	$3/5 \div 7/8 = \frac{3/5}{7/8} \times 1 = \frac{3/5}{7/8} \times \frac{8/7}{8/7}$ $= \frac{3/5 \times 8/7}{7/8 \times 8/7} = \frac{3/5 \times 8/7}{1} =$ 3/5 x 8/7 = 24/35

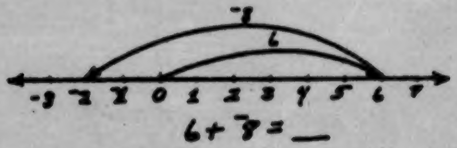
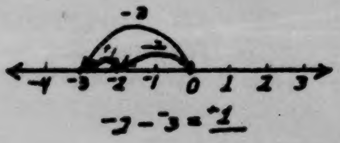
Major Topic	Behavioral Objectives	Teaching Aids
Algorithms (continued) Division	(h) The student can name the quotients (as fractions in lowest terms and/or as mixed numerals) for problems such as: 3 1/2 ÷ 4/7 = _____ and 7 2/3 ÷ 5 1/8 = _____ by renaming as improper fractions and dividing.	
4. Properties of Multiplication and Division		
Properties Commutative and Associative Properties of of Multiplication	(a) The student can solve equations such as: 1/2 x 3/4 = 3/4 x _____ (1/2 x 3/4) x 4/7 = 1/2 x (3/4 x _____)	
	(b) Given a multiplication problem with two or more factors, the student can demonstrate how to find the product in the easiest way by rearranging the factors.	$\frac{14}{15} \times \frac{12}{35} = \frac{14 \times 12}{15 \times 35} = \frac{14 \times 12}{35 \times 15} =$ or $\frac{14}{35} \times \frac{12}{15} = \frac{2}{5} \times \frac{4}{5} = \frac{8}{25}$
Identity Element for Multiplication (1=1/1=2/2=3/3...)	(c) The student can solve equations such as: 5/5 x 3/7 = _____ 5/6 x 13/13 = _____ 3/4 x _____ = 3/4 2/3 x _____ = 1	
	(d) The student can demonstrate how to rename a given fraction such as 2/3 by multiplying by some name for the identity element.	2/3 x 2/2 = 4/6 2/3 x 3/3 = 6/9 2/3 x 4/4 = 8/12

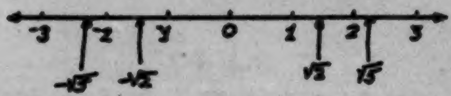
Major Topic	Behavioral Objectives	Teaching Aids
Multiplication Inverses (reciprocals)	(e) Given a set of rational numbers such as 2/3, 4, 7, 7/4, 1, and 0, the student can name their multiplication inverses (reciprocals) and demonstrate that the product of any rational number and its reciprocal is 1.	2/3 x 3/2 = 1, 4 x 1/4 = 1, 7/4 x 4/7 = 1, 1 x 1 = 1, 0 x ? = 1 ∴ 0 has no reciprocal
Multiplication Property of 0 (0=0/1=0/2=0/3...)	(f) The student can solve equations such as: 2/3 x _____ = 0/5 0 x 2/3 = _____ 4/5 x 0/13 x 9/17 = _____	
	(g) The student can demonstrate that 0/0 does *not* name a unique rational number.	0 - 0 = 0 since 0 x 0 = 0 0 - 0 = 3 since 3 x 0 = 0 0 - 0 = 17/35 since 17/35 x 0 = 0
Distributive Property of Multiplication over addition	(h) Given a problem such as 5 1/2 x 3 = _____, the student can demonstrate his understanding of the distributive principle by multiplying in horizontal form.	5 1/2x3 = (5+1/2)x 3 = (5x3) + (1/2x3) = 15 + 1 1/2 = 16 1/2
5. Operations with Decimals		
Decimal Notation (including money)	(a) Given any "reasonable" addition or subtraction problem, the student can name the sum or difference. (b) The student can name the products and quotients for problems such as: 21.07 x 8.4 $25\overline{)\ 43.75}$ NOTE: In division problems, use only whole number divisors.	

NUMBER AND OPERATIONS

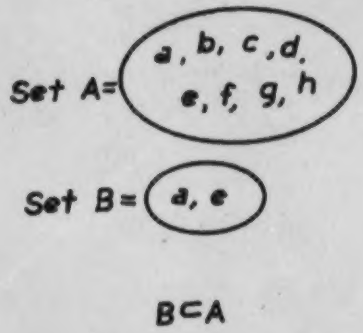
GRADE SIX

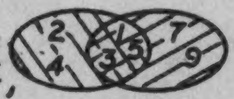
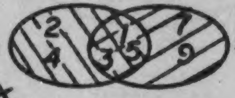
Major Topic	Behavioral Objectives	Teaching Aids
E. THE INTEGERS		
1. Ordered Pairs	Given $x \in \{0, 1, {}^{-}2\}$ and $y \in \{{}^{-}1, 2, 3\}$, the student must be able to tabulate the set of ordered pairs (x, y) and the set of ordered pairs (y, x). Example: $(x, y) \in \{(0, {}^{-}1), (0, 2), (0, 3), (1, {}^{-}1), (1, 2), (1, 3), ({}^{-}2, {}^{-}1), ({}^{-}2, 2), ({}^{-}2, 3)\}$ $(y, x) \in \{({}^{-}1, 0), ({}^{-}1, 1), ({}^{-}1, {}^{-}2), (2, 0), (2, 1), (2, {}^{-}2), (3, 0), (3, 1), (3, {}^{-}2)\}$	x 0, 1, ${}^{-}2$ is read "x is an element of the set containing 0, 1 and ${}^{-}2$." (x, y) $(0, {}^{-}1), (0, 2)$ is read "the ordered pair (x, y) is an element of the set containing the ordered pairs $(0, {}^{-}1)$ and $(0, 2)$."
2. Coordinate Axes	Given the set of ordered pairs $\{(0, {}^{-}1), (0, 2), (0, 3), (1, {}^{-}1), (1, 2), (1, 3), (2, 1), (2, 2), ({}^{-}2, 3)\}$ The student must be able to correctly locate the points on a coordinate axis, for instance, Point A is 2 over to the right the origin (*crossing point of the axes*) and 3 up. Since both numbers are positive, the coordinates are (2, 3). Point B is right 1 down 2, hence the ordered paid is $(1, {}^{-}2)$.  3 2 1 -1 -2 -3 -3 -2 -1 1 2 3 4 A B C D C is $({}^{-}2, {}^{-}1)$ D is $({}^{-}3, 2)$	Many examples can be made to illustrate the *Cross Product* asked for in 1-(a). For example: Match the names of the set of boys John, Bob, Richard with the names of the set of girls Carol, Margaret, Joy in all possible ways in which the boys could ask the girls for dances at a party, i.e. (John, Carol), (John, Margaret), (John, Joy), (Bob, Carol), (Bob, Margaret), (Bob, Joy), (Richard, Carol), (Richard, Margaret), (Richard, Joy) .

Major Topic	Behavioral Objectives	Teaching Aids
E. THE INTEGERS (continued)		
3. The Negative Rationals		
Directed Numbers	(a) Given the numbers ${}^{+}3$, ${}^{-}3$, ${}^{+}4$, ${}^{-}2$, 0, the student can graph them on the number line.	Refer to Grade V, Section E-3 (a).
	(b) Given a numeral such as ${}^{-}8$, the student can name it as negative eight or the opposite of eight.	
4. Addition and Subtraction of Integers		
Definition of Addition	(a) Given a number line model for the addition of two integers, the student can determine and name the sum.	 ${}^{-}8$ 6 -3 ${}^{-}2$ ${}^{-}1$ 0 1 2 3 4 5 6 7 $6 + {}^{-}8 =$ ___
	(b) Given an addition problem such as: ${}^{-}5 + 13 =$ _____, the student can name the sum and demonstrate how to find the sum by using a number line.	
Definition of Subtraction	(c) Given a subtraction equation, such as: ${}^{-}2 - {}^{-}3 =$ _____, the student can write and solve the related addition equation, _____ $+ {}^{-}3 = {}^{-}2$.  -3 -1 -4 -3 -2 ${}^{-}1$ 0 1 2 3 ${}^{-}2 - {}^{-}3 = \underline{{}^{+}1}$	Example: ${}^{-}2 - {}^{-}3 = \underline{{}^{+}1}$ because $\underline{{}^{+}1} + {}^{-}3 = {}^{-}2$ Since subtraction is the inverse of addition, start the drawing of the second arrow from where the other arrow began rather than from where the arrow ended as in addition. Now count from the point where the second arrow ended to the point where the first arrow ended, noting the direction for the sign of the answer.

Major Topic	Behavioral Objectives	Teaching Aids
5. Multiplication and Division of Integers	(a) Given a set of integers, having less than two digits, the student must be able to determine the product of any two. (b) Given a set of integers having less than two digits, the student must be able to determine the quotient of any two that are evenly divisible.	
F. THE IRRATIONAL NUMBERS		
1. The Number Line	Given an integral number line and $\{-\sqrt{5}, -\sqrt{2}, \sqrt{2}, \sqrt{5}\}$, the student must be able to correctly approximate the points associated with the members of the set between the integers.	 ⁻3 ⁻2 ⁻1 0 1 2 3 $-\sqrt{5}$ $-\sqrt{2}$ $\sqrt{2}$ $\sqrt{5}$

SETS, FUNCTIONS, RELATIONS, SYSTEMS, AND LOGIC GRADE SIX

Major Topic	Behavioral Objectives	Teaching Aids
A. SETS		
1. Collections	Given a description of a set, the student distinguish between members of the set and things that are not members.	For example: Given the set of whole numbers, the student can list the set of even numbers less than 20.
2. Equivalent and Non-Equivalent Sets	(a) Given two sets, the student can identify them as equivalent or non-equivalent. If they are equivalent the student can form a one-to-one correspondence between the sets.	For example: Given the set of students in the room and the set of desks.
Subsets	(b) Given a set, the student can identify and describe a subset of the given set.	
3. Cardinal Number 0 - Infinity	The student can determine the cardinal number of a given set.	
4. Commutative, Associative, and Distributive Property	The student shall be able to demonstrate the commutative, associative, and distributive properties through the use of sets.	See previous grades for illustrations.

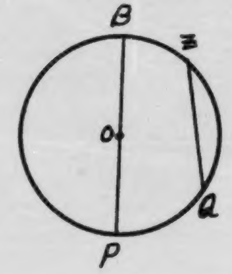
Major Topic	Behavioral Objectives	Teaching Aids
5. Equal Sets	Given two sets, the student can identify them as equal or not equal.	Set A = {△, ⊖, ▢} Set B = {⊖, △, ▢} Set C = {⊖} Set A = Set B; Set A ≠ Set C
6. Union and Intersection of Sets	Given two sets (disjoint or not disjoint), the student can give the set that is the union and the set that is the intersection of the two sets.	Set D = {1,2,3,4,5} Set E = {1,3,5,7,9} D∪E =  = {1,2,3,4,5,7,9} \\ & //, + # D∩E =  = {1,3,5} only #
7. Finite and Infinite	(a) Given a list of finite and infinite sets, the student can identify those which are finite and those which are infinite.	The set of numbers of arithmetic is an infinite set, while the set of whole numbers between 10 and 20 is finite.

Major Topic	Behavioral Objectives	Teaching Aids
7. (continued)		
Empty or Not Empty	(b) Given a list of sets, the student can tell whether they are empty or non-empty.	For example: The set of whole numbers greater than four is a non-empty set. The set of even numbers that divide 11 exactly is an empty set.
8. Venn Diagrams	The student can construct Venn diagrams to show the union and intersection of two sets.	See Grade VI, Section A-6 (a) for diagrams.
9. Set Symbolism	(a) The student shall be able to show set union and intersection using set notation.	$F = \{M, N, X\}$; $G = \{R, X, T\}$ $F \cup G = \{M, N, R, X, T\}$; $F \cap G = \{X\}$
	(b) The student shall be able to show one set is or is not a subset of another using set notation.	Set A = $\{a, b, c, d, e\}$ Set B = $\{b, c, d\}$; Set C = $\{f, g, h, i\}$ Set B is a subset of Set A, $B \subset A$ Set C is not a subset of Set A, $C \not\subset A$
	(c) The student can give the number of a set using set notation.	n(A) = 5 from above
	(d) The student can express finite and infinite sets using set notation.	Example: The set of whole numbers between 0 and 100. $\{1, 2, 3, \ldots 99\}$ The set of whole numbers greater than 50. $\{51, 52, 53, \ldots\}$

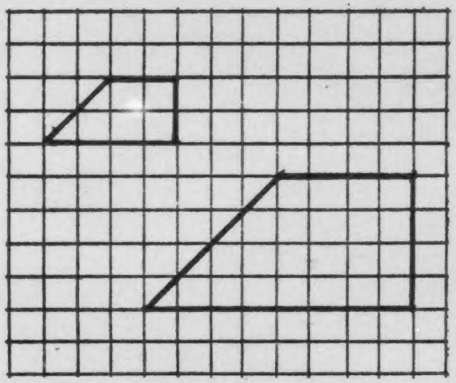
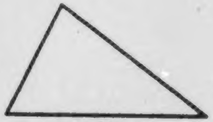
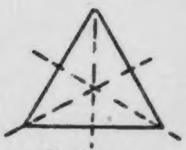
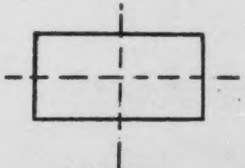
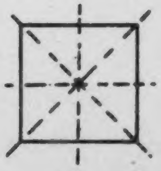
<table>
<tr><th>Major Topic</th><th>Behavioral Objectives</th><th>Teaching Aids</th></tr>
<tr><td>B. RELATIONS AND FUNCTIONS</td><td></td><td></td></tr>
<tr><td>1. "What's My Rule?"</td><td>Given the game situation described in the addenda, the student shall be able to determine the missing values, and state the rule of the game.</td><td>See addenda.
<table>
<tr><th>In</th><th>Out</th></tr>
<tr><td>(4,5)</td><td>31</td></tr>
<tr><td>(6,2)</td><td>42</td></tr>
<tr><td>(5,4)</td><td>37</td></tr>
<tr><td>(2,3)</td><td>?</td></tr>
</table>
<table>
<tr><th>In</th><th>Out</th></tr>
<tr><td>2</td><td>7</td></tr>
<tr><td>5</td><td>10</td></tr>
<tr><td>6</td><td>11</td></tr>
<tr><td>8</td><td>?</td></tr>
</table>
</td></tr>
<tr><td>2. Number Patterns</td><td>The student shall be able to complete the next few values in number patterns.
$\{4, 5, 7, 11, 19, \_\_\_, \_\_\_, \_\_\_, \ldots\}$
$\{14, 16, 20, 26, \_\_\_, \_\_\_, \_\_\_, \ldots\}$</td><td>See addenda.</td></tr>
<tr><td>C. MATHEMATICAL SENTENCES</td><td></td><td></td></tr>
<tr><td>1. Open, True, or False Sentences</td><td>(a) The student can classify mathematical sentences as open, true or false.</td><td>$\square + \triangle < 8$; $9 \div 2 = \square$;
$3\text{-}1/2 \times 2 = 7$; $5 - 3 = 9$</td></tr>
<tr><td></td><td>(b) The student can use the "is not equal to" sign, the "less than" and the "greater than" signs in writing mathematical sentences.</td><td>$\frac{3}{8} \neq \frac{7}{9}$; $4 > 5 - 3$</td></tr>
<tr><td></td><td>(c) The student can find the solution to open sentences that involve negative numbers.</td><td>$^{-}5 + {}^{-}8 = \square$</td></tr>
</table>

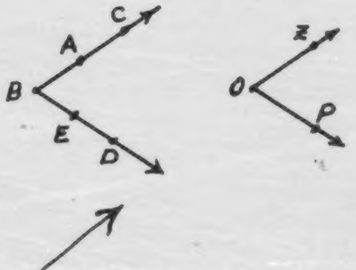
Major Topic	Behavioral Objectives	Teaching Aids
C. MATHEMATICAL SENTENCES (continued)		
	(d) The student can find the solution set to mathematical sentences that involve multiplication and division of fractional numbers and decimals.	$\frac{\square}{4} \times \frac{1}{5} = \frac{3}{20}$; $\frac{3}{8} \div \frac{5}{4} = \frac{\triangle}{\square}$
	(e) The student can translate English sentences into mathematical sentences.	For example: If one meter of material cost $2.98, what would be the cost of one-half meter of the same material? $1/2\ n = \$2.98$
	(f) The student shall be able to use parenthesis to show order of operation in mathematical expressions involving addition, subtraction, multiplication, and division.	$(9 \div 3) + (7 \times 8) - (4 + 2)$

Major Topic	Behavioral Objectives	Teaching Aids
A. RECOGNITION AND DESCRIPTION OF SOLID FIGURES		
1. Solid Figures		
Content Point Plane Polyhedron prism pyramid Sphere hemisphere Cylinder Cone	Given models of the space figures named on the left (wood or plastic solids, paper models, sketches, etc.), the student can *identify, name,* and *distinguish* among them.	
B. RECOGNITION AND DESCRIPTION OF PLANE FIGURES (as sets of points)		
1. Plane Figures		
Point Path (curve) Line Line Segment Ray	(a) Given models of the plane geometric figures named on the left (wire, paper or flannel cutouts, pencil or chalk outlines), the pupil can *identify, name,* and *distinguish* among them.	

Major Topic	Behavioral Objectives	Teaching Aids
1. Plane Figures (continued)		
Lines Parallel Perpendicular	(b) The student can *read* and *write standard notation* for the plane figures named on the left. (NOTE: See notation beside names on the left; also, figure and notation to the right.	Circle Center  Radii $\overline{BO}$ and $\overline{PO}$ Diameter $\overline{BP}$ Chord $\overline{QZ}$ Arc $\overset{\frown}{ZQ}$
Angle $\angle ABC$ Right Acute Obtuse Polygon Triangle $\triangle ABC$ right triangle Quadrilateral (diagonal) parallelogram square rectangle		
Circle Center Radius Diameter Chord Circumference arc		

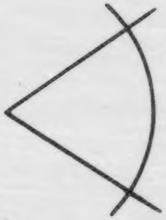
Major Topic	Behavioral Objectives	Teaching Aids
C. NAME OF FIGURES 1. Solid Figures Vertex Edge Lateral Surface (face) Base Altitude (height)	(a) Given a set of space figures, the student can *identify* and *name* the various parts of of each as plane figures.	For Example: The lateral surfaces (faces) of a right rectangular prism are rectangles. The bases of a cylinder are circles.
	(b) The student can describe a given plane figure as a *set of points*.	For example: A pentagon is the set of points in a simple closed curve composed of the union of five line segments.
	(c) The student can describe a given space figure as a *set of points*.	For example: A sphere is the set of points one inch from a given point in space.
	(d) The student can *draw* and *name* a set of points satisfying given conditions. For example: The set of all points in space one inch from a given line is a *(n) cylindrical* surface. A right cylinder is formed when two parallel planes intersect a cylindrical surface at right angles to the given line in the example. The two circles (sections) in the planes and the part of the cylindrical surface between the two planes together form the cylinder.	

Major Topic	Behavioral Objectives	Teaching Aids
D. RELATIONS OF GEOMETRIC FIGURES		
1. Similarity	(a) Student, when given the dimensions of a figure on squared paper, will double each dimension to produce an enlarged similar figure.	Students may be introduced to the pantograph to deepen understanding of enlarging irregular shapes. Diagram shown below: 
	(b) Given a mirror and drawings of geometric figures, the student will determine if the figures have symmetry and if they do, will draw the lines of symmetry on the figure.	 no lines  3 lines  2 lines  4 lines

Major Topic	Behavioral Objectives	Teaching Aids
2. Congruency	(a) Given a pair of line segments, angles, triangles, or other polygons, the student can *identify* the pairs as congruent or *not* congruent by matching the figures in some manner (trace and overlay, cutouts, etc.).	
	(b) The student can use the symbol to *express the relationship* between congruent figures. ($\cong$).	
	(c) The student can *distinguish* between equal and congruent figures.	
	(d) Given two congruent triangles, the student can *name* the pairs of congruent line segments and the pairs of congruent angles.	$\left\{\begin{array}{l}\angle ABE = \angle CBD \\ \angle ABE \cong \angle ZOP\end{array}\right.$
E. CONSTRUCTION - APPROXIMATE AND EXACT		
1. Properties Copy: Line Segment Angle Triangle	(a) The student can *sketch, describe,* and *give* examples of parallel, intersecting, and perpendicular lines.	See Grade V, Section C., 1. (a,b,c).
	(b) Using a straightedge and compass, the student can construct a plane figure *congruent* to a given line segment, angle or triangle.	See Grade V, Section C., 1. (e).

GEOMETRY GRADE SIX

Major Topic	Behavioral Objectives	Teaching Aids
1. Properties (continued)		
Angle Bisector	(c) Using a straightedge and compass, the stu- can construct the bisector of a given angle.	
Perpendicular to a Line Segment	(d) Using a straightedge and compass, the student can construct the perpendicular bisector of a given line segment.	For example: See Grade V, Section C., 1. (f).
Perpendicular to a Line at a Point on the Line	(e) Given a line and a point of the line, the student can construct a line perpendicular to the given line at the given point.	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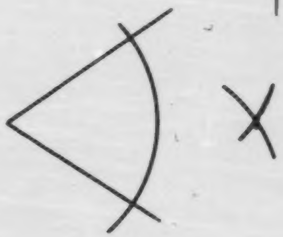
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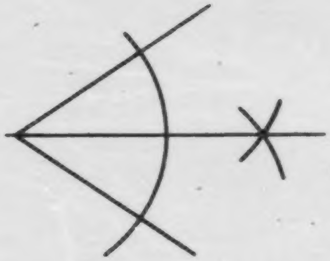
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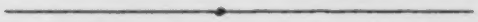

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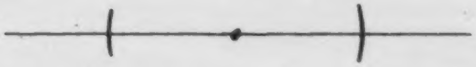

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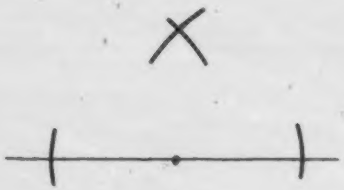

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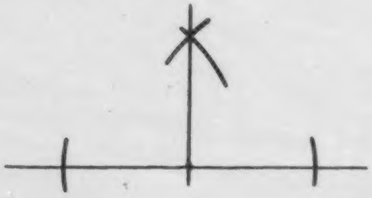
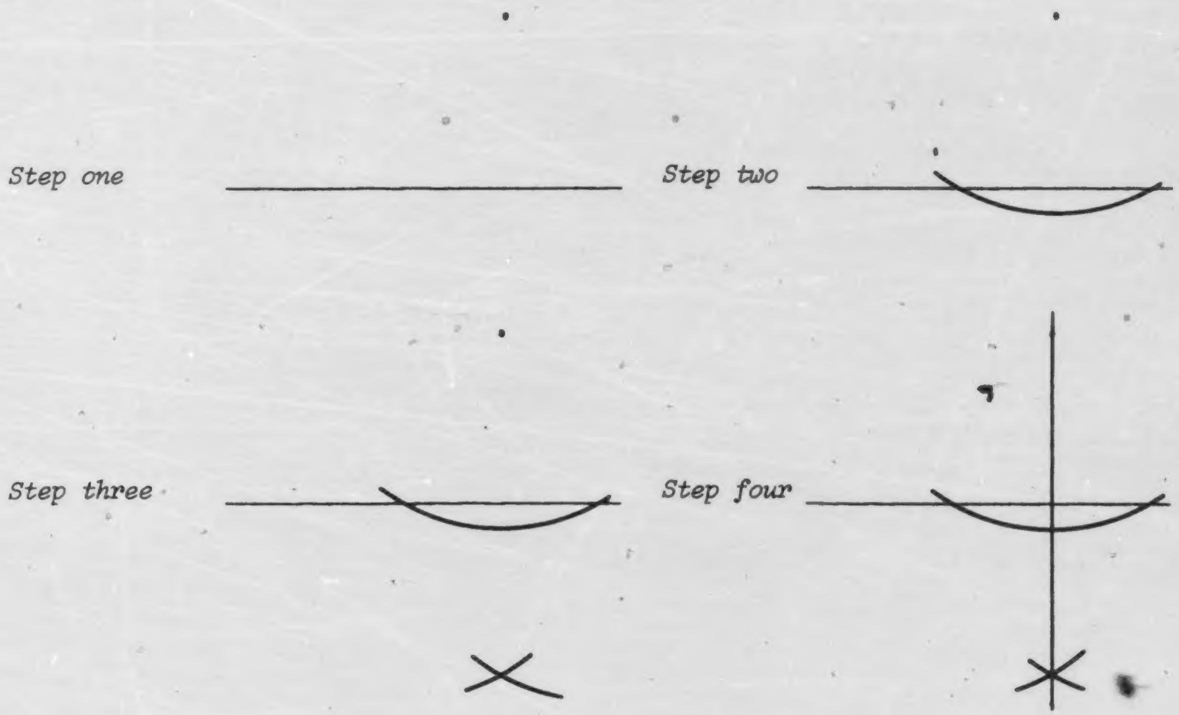
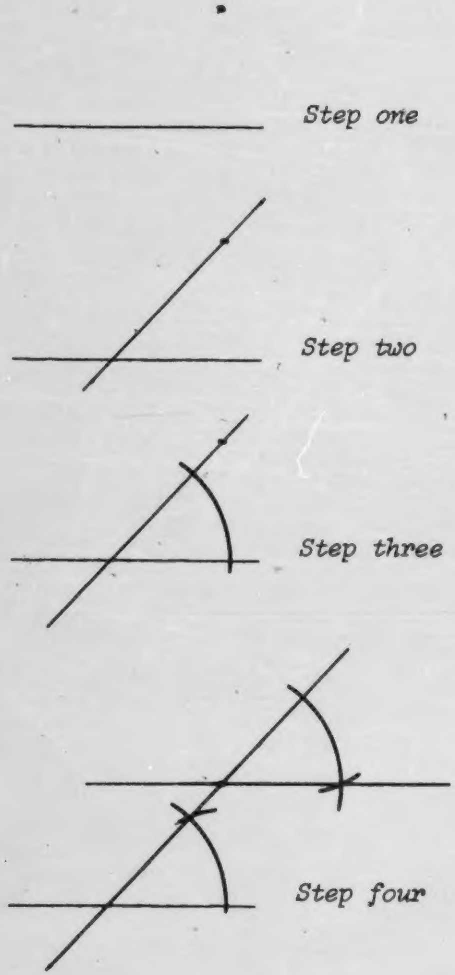


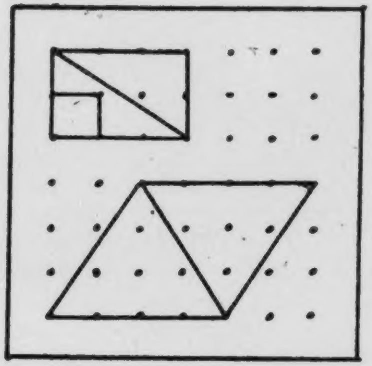
Major Topic	Behavioral Objectives	Teaching Aids
1. Properties (continued)		
Perpendicular to a Line from a Point *not* on the line	(f) Given a line and a point *not* on the line, the student can construct a line perpendicular to the given line through the given point.	


Step one
Step two
Step three
Step four

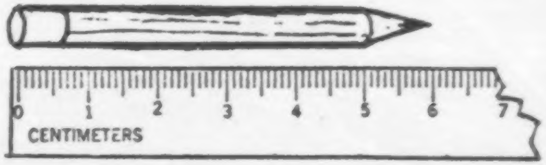
Major Topic	Behavioral Objectives	Teaching Aids
1. Properties (continued)		
Line Parallel to a Given Line	(g) Given a line and a point *not* on the line, the student can construct a line parallel to the given line through the given point.	
	(h) Using a compass, the student can construct a circle with a given center and radius (or diameter).	

Major Topic	Behavioral Objectives	Teaching Aids
A. CONCEPT OF MEASUREMENT		
1. Process of Measuring	(a) Given a measurable physical property such as length, area, weight, temperature, etc., the student can select a suitable unit and/or measuring device and measure the property. For example: An angle can be measured by using a (*n*) ____________________. What is an appropriate unit when discussing the weight of diamonds? ________________________. Of coal?______. Of a human being?______.	Use material from the laboratory and history of measurement. Let students invent methods of measurements and then discuss the advantages and disadvantages of their units of measure. Have tables of equivalent measure available for the students.
Arbitrary Selection of Unit	(b) The student can name at least two units suitable for naming the measure of a given physical property. For example: Speed can be expressed in kilometers per hour or in meters per second.	
Approximate Nature of Measurement	(c) The student can demonstrate his understanding of the approximate nature of measurement by stating the precision of the measure. For example: The circumference of the earth at the equator is 41,000 kilometers correct to the nearest thousand kilometers. The thickness of a piece of paper is .008 centimeters correct to the nearest thousandth of a centimeter.	Have several students measure the same distance and record the measurement on a slip of paper. Collect the papers, and ask if the students all have the same measurement. Discuss the results.

Major Topic	Behavioral Objectives	Teaching Aids
1. Process of Measuring (continued)	(d) Given that the measures of a line segment is 3 1/2 centimeters correct to the nearest half centimeter, the student can state that the actual length is between 3.25 and 3.75 centimeters.	
2. Concept of Graphs	The student can plot points correctly on a coordinate graph.	Use a collection of different sets of graphs. Have students bring graphs from papers and magazines. Play the game of Tic Tac Toe. See addenda.
B. MEASUREMENT OF PHYSICAL PROPERTIES		
1. Time Year, Decade, Century, B.C. or A.D.	(a) Given that a man was born in 19 B.C. and died in 42 A.D., the student can compute the age of the man.	Use headline stories.
Time Zones	(b) Given that a football games begins at 1:00 p.m. Eastern Standard Time, the student can name the time to watch it live on television in his home town.	Find maps with time zones marked; bus, train, and airline time tables.
2. Money Percent	The student can find the interest or commission on a given amount.	Use items from newspapers and have the students figure the amount of savings. Figure the interest on money in savings accounts.
3. Linear Measurement Metric Units Light Years	(a) The student can use various measuring devices (ruler, meter stick, tape) to measure length in whole and fractional parts of units.	Use laboratory material. See addenda.

Major Topic	Behavioral Objectives	Teaching Aids
Perimeter	(b) Given the measures of the sides of a polygon, the student can compute the perimeter.	Have geo-board, grid paper, and large cardboard polygons for the students to measure and figure the perimeter.
Circumference	(c) Given the measure of the diameter or radius of a circle, the student can compute the compute the circumference (perimeter).	Use trundle wheel measuring device. See addenda.
4. Area Square Units Related to Units of Length (acre) Rectangle Parallelogram Triangle Circle	(a) Using paper figures and scissors, the student can demonstrate that the area of a triangle is equal to one-half of the area of a parallelogram having the same base and altitude measures.	Use geo-board, clear plastic overlays, and checkerboard grid or pegboard. 
	(b) Given the measures of the base and altitude of a rectangle, parallelogram or triangle, the student can compute the area.	
	(c) Given the measure of the diameter or radius of a circle, the student can compute the area.	
	(d) The student can compute the surface area of space figures whose faces are rectangles, parallelograms or triangles.	
5. Volume Cubic Units Related to Units of Length	(a) Given the measures of the edge of a right rectangular prism, the student can compute the volume.	Build units of measure out of construction paper or use their own unit of measure. How many marbles does it take to fill the box?
Liquid Measure Metric Units	(b) The student can name the common standard units of liquid measure and measure the capacity of a given container to the nearest whole unit.	Use laboratory materials. See Grade V, Section B., 5. (a). See addenda.

Major Topic	Behavioral Objectives	Teaching Aids
6. Weight-Mass	The student can name the common metric units of weight (mass) measure and can name quantities that would be measured by them.	Use laboratory materials. See addenda.
7. Temperature Degrees Celsius	Using a thermometer calibrated in degrees Celsius the student can read the temperature to the nearest degree.	Use different types of thermometers.
8. Angle Degree	Using a protractor, the student can measure an angle to the nearest degree.	Use large wooden or cardboard demonstration protractor.
9. Speed meters/second kilometers/hour	(a) The student can read a speedometer. (b) Given a distance and the time necessary for a car to travel the distance, the student can compute the average speed of the car.	Ask question such as: "If you travel 40 kilometers per hour for two hours, how far would you go? If you travel 300 kilometers in 5 hours, what was the average speed of the car?"
10. Estimation with Inequalities cm/centimeters	The student can estimate answers to problems like the following: The thickest book in a set of encyclopedias is 3.2 cm. The thinnest book is 1.9 cm. Estimate the length of shelf needed to hold the complete set of 20 books.	Have students make up estimation problems of their own. Count the books on one shelf of a bookcase then estimate the number of books in a bookcase.
11. Graphs	The student can make circle and divided bar graphs based on 100 percent.	Have students collect data and then decide what kind of graph would best illustrate the information collected.

Major Topic	Behavioral Objectives	Teaching Aids
12. Accuracy and Precision in Measurements	(a) If the measurement of a segment is 3.25 cm, the measurement is precise to ____ and the greatest possible error is ____.	Have students measure objects and tell how accurate their measurement are. Have them state the precision and the greatest possible error for each measurement. 
Indirect Measurement	(b) The student can estimate the height of the flagpole or height of the wall.	Use of scale drawings to make comparison.
13. Statistics	(a) The student understands the meaning of data and can use data in making graphs and performing computational problems.	Have the students collect data. Let them ask their own questions about the data they collected.
Averaging	(b) Given a set of numbers such as 98, 75, 83, 100, and 70, the student can name the average (arithmetic mean) of the numbers.	
C. RENAMING MEASURES		
1. Comparison of Units Conversion of Units	The student can express the relationships between units of measure appropriate to the grade level and can rename a measure in other units. Example: 1 cubic decimeter = __ cubic cm	

Major Topic	Behavioral Objectives	Teaching Aids
D. COMPUTATIONS WITH MEASURES	The student can compute with measures appropriate to the grade level, assign the proper unit to the result, and rename if necessary. For example: If it takes 8 hours to travel 2880 kilometers then km per hour = __________ $8\text{ hrs.}\overline{)\,2880\text{ km}}$ km per minute =	

Major Topic	Behavioral Objectives	Teaching Aids
A. THE NATURAL NUMBERS - COUNTING AND NUMERATION		
1. Primes and Composites	Given any whole number having less than four digits, the student must be able to determine its complete factorization.	Blackboard, overhead projector, tapes, records. Using the list of prime numbers as possible divisors, some students may find the following method of complete factorization helpful. For example: 2)870 ∴870 = 2 x 3 x 5 x 29 3)435 5)145 29
2. Exponential Notation	(a) The student must be able to write any number, having less than seven digits, in expanded notation using exponents.	Students should be led to discover that they need to try only the prime numbers as divisors that are less than the square root of the number to be factored.
	(b) The student must be able to write any number, having less than seven digits, in scientific notation.	
B. THE NATURAL NUMBERS - OPERATIONS		
1. Greatest Common Factor - Least Common Multiple	(a) Given a set of three numbers having less than four digits, the student must be able to determine the greatest common factor. Students should see the application of G.C.F. in reducing fractions to lowest terms.	Some students may find the method of aligning or "matching" common prime factors helpful in determining the greatest common factor. For example; 154 = 2 x 7 x 11 G.C.F. = 2 x 11 374 = 2 x 11 x 17 = 22 572 = 2 x 2 x 11 x 13

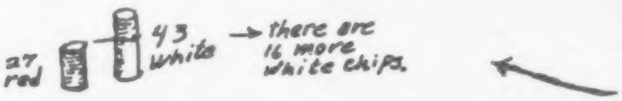
Major Topic	Behavioral Objectives	Teaching Aids
1. Greatest Common Factor - Least Common Multiple (continued)	(b) Given a set of the numbers having less than four digits, the student must be able to determine the least common multiple. Students should see the application of L.C.M. in finding the lowest common denominator in adding and subtracting fractions.	Also some students may find circling every different factor the greatest number of times it appears in any factorization helpful in determining the least common multiple. For example: $180 = 2 \times 2 \times \boxed{3 \times 3} \times 5$ $220 = 2 \times 2 \times 5 \times \boxed{11}$ $400 = \boxed{2 \times 2 \times 2 \times 2} \times \boxed{5 \times 5}$ $\text{L.C.M.} = \overbrace{2 \times 2 \times 2 \times 2} \times \overbrace{3 \times 3} \times \overbrace{5 \times 5} \times \overbrace{11} = 39{,}600$
C. THE RATIONAL NUMBERS		
1. Equivalent Fractions and Inequalities	Given a set of rational numbers, the student must be able to determine whether any two are equivalent, or if they are not, they must determine which is less and which is greater.	Multiplication Table - see addenda, see Teaching Aids.
2. Ordering Fractions	Given a set of ten fractions, the student must be able to order them from least to greatest.	Multiplication Table - see addenda. Several ways to determine equivalency and order of fractions. For example: 2/3, 5/8, 11/18, 4/7 (continued on next page)

Major Topic	Behavioral Objectives	Teaching Aids

	Equivalent Decimal	Numerator Test	Denominator Test
2/3	.66...	$\frac{220}{330}$	$\frac{338}{504}$
5/8	.625	$\frac{220}{352}$	$\frac{315}{504}$
11/18	.611	$\frac{220}{360}$	$\frac{308}{504}$
4/7	.571428	$\frac{220}{385}$	$\frac{288}{504}$

Major Topic	Behavioral Objectives	Teaching Aids
3. Number Pairs, Ratio and Proportion	Given a proportion of the type 2/3 = x/9, the student should be able to determine the value of the missing component.	
4. Decimal Fractions	(a) Given a set of terminating decimal fractions, the student should be able to determine the equivalent common fractions. (b) Given a set of common fractions, the student should be able to determine the equivalent decimal fractions.	
D. THE RATIONAL NUMBERS - OPERATIONS		
1. Addition and Subtraction of Fractions	Given fractions of the type 37/39 and 19/48, the student should be able to determine the sum and difference.	The student can use his/her ability to find the least common multiple to determine the lowest common denominator. Multiplication Table - see addenda.

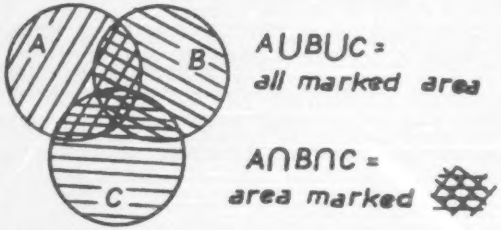
Major Topic	Behavioral Objectives	Teaching Aids
2. Properties of Addition and Subtraction	Given equations of the following type, the student should be able to solve them: (1) $\frac{3}{7} + \frac{6}{15} = \frac{6}{15} + \square$ (2) $\frac{9}{16} + \frac{3}{4} + \frac{5}{9} = \left(\square + \frac{3}{4}\right) + \frac{5}{9}$ (3) $\frac{17}{21} + \frac{0}{21} = \square$	
3. Multiplication and Division of Fractions	Given fractions of the type $\frac{16}{39}$ and $\frac{78}{25}$, the student should be able to determine the product and quotient.	The student can use his/her ability to find the greatest common factor to reduce products and quotients to lowest terms.
4. Properties of Multiplication and Division	Given equations of the following type, the student should be able to solve them: (1) $3 \cdot 5 = 5 \cdot (\_\_\_)$. (2) $\frac{5}{7} \cdot \frac{4}{7} \cdot \frac{5}{3} = \left(\frac{5}{7} \cdot \square\right) \cdot \frac{5}{3}$. (3) $\frac{13}{27} \cdot \frac{27}{27} = \square$ (4) $\frac{15}{19} \cdot \square = \frac{15}{19}$	

Major Topic	Behavioral Objectives	Teaching Aids
E. THE INTEGERS		
1. Properties of Naturals Extended to Integers	Given equations of the following type, the student should be able to solve them: (1) $^{+}7 + {}^{-}9 = {}^{-}9 + \_\_\_\_$ (2) $^{-}3 \cdot ({}^{+}4) = {}^{+}4 \cdot (\_\_\_\_)$ (3) $^{+}9 + ({}^{-}3 + {}^{-}5) = ({}^{+}9 + \_\_\_\_) + {}^{-}5$ (4) $^{-}7 \cdot (\_\_\_\_ \cdot {}^{-}3) = ({}^{-}7 \cdot {}^{+}4) \cdot {}^{-}3$ (5) $^{-}3 \cdot ({}^{+}4 + {}^{-}7) = {}^{-}3 \cdot ({}^{+}4) + {}^{-}3 \cdot (\_\_\_\_)$ (6) $^{-}5 + \_\_\_\_ = {}^{-}5$ (7) $^{-}3 \cdot (\_\_\_\_) = {}^{-}3$	
2. Addition and Subtraction of Integers	Given problems of the following type, the student should be able to solve for the variable. (1) $^{+}27 + {}^{-}43 = X$ (2) $^{-}93 - {}^{-}47 = X$  27 red 43 white → there are 16 more white chips.	Use horizontal and vertical number-lines. See Addition Matrix in the addenda. Use non mathematical models such as a helicopter (+) and submarine (-) to help students visualize opposites. Another of these models is to walk forward (+), to walk backward (-), not walk forward (-+), not walk backward (--). Use different colored chips to represent positive and negative numbers. For example: red chips are representing (+) - white chips are representing negative (-). Which are there more of?

Major Topic	Behavioral Objectives	Teaching Aids
3. Multiplication and Division of Integers	Given problems of the following type, the student should be able to solve for the variable. (1) $^{-}12 \cdot (^{+}7) = X$ (2) $^{-}144 \div (^{-}16) = X$	See addendum on "A Rationale in Working with Signed Numbers." Multiplication Matrix, "A Rationale in Working with Signed Numbers - Revisited."

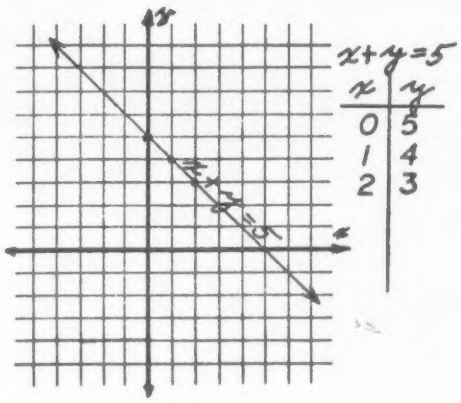
SETS, FUNCTIONS, RELATIONS, SYSTEMS, AND LOGIC GRADE SEVEN

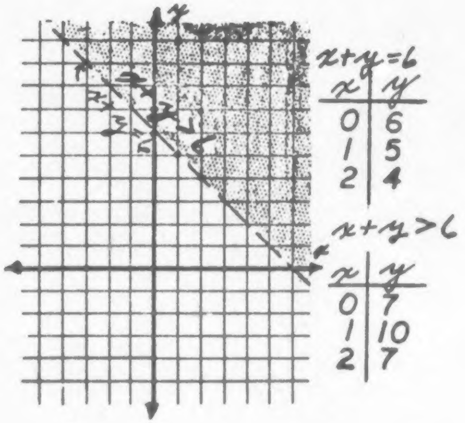
Major Topic	Behavioral Objectives	Teaching Aids
A. SETS		
1. Collections	(a) Given a description of a set, the student can list the members of the set using the roster form.	For example: Set A contains all even numbers between 15 and 25. Set A = {14, 16, 18, 20, 22, 24}
	(b) Given a set in roster form, the student can give a description of the set.	Set B = {2, 3, 5, 7, 11} Set B contains the prime numbers less than 12.
Elements	(c) Given a description of a set, the student tell the members of the set and those that are not members of the set.	
2. Equivalent and Non-equivalent, Equal and Non-equal Sets	(a) The student can compare sets and tell whether they are equivalent or non-equivalent. If they are equivalent, he/she can tell whether the sets are equal or not equal.	
	(b) The student can form one-to-one or one-to-many correspondence between the members of two sets.	
Subsets Proper	(c) The student can complete questions, such as the following pertaining to sets and subsets:	
	(*i*) If each member of one set is also a member of a second set, then the first set is a ______ of the second set.	*i* subset
	(*ii*) A ______ ______ is a subset that contains some but not all members of the original set.	*ii* proper subset

Major Topic	Behavioral Objectives	Teaching Aids
3. Cardinal Number 0 - Infinity	The student can determine the cardinal number of a set.	Refer to Grades I - VI for examples and more information.
4. Commutative, Associative, and Distributive Property	The student can use sets to illustrate the commutative, associative, and distribute properties.	
5. Union and Intersection of Sets	Given two sets, the student can form the union and the intersection.	
6. Finite and Infinite Empty and Non-empty	Given a list of sets, the student can classify them as finite or infinite; empty or non-empty.	
7. Venn Diagrams	The student can use Venn diagrams to show the union and intersection of two or more sets.	 A B C $A \cup B \cup C$ = all marked area $A \cap B \cap C$ = area marked

Major Topic	Behavioral Objectives	Teaching Aids
8. Set Symbolism	The student can use the following set notation correctly: $\{\ \}$, $\supset$, $\cup$, $\cap$, $N(\ )$	For example: Set A = $\{a, b, c\}$; Set B = $\{d, e, f\}$ $n(A) = 3$ $A \not\subset B$; $A \cap B = \emptyset$ $A \cup B = \{a, b, c, d, e, f\}$
B. RELATIONS AND FUNCTIONS		
1. "What's My Rule?"	Given the game situation described in the addenda, the student shall be able to determine the missing values, and state the rule of the game.	See addenda. Also see previous grades under "What's My Rule?"
2. Number Patterns	The student shall be able to complete the next few values in number patterns. $\{2, 4, 8, 16, \_\_\_, \_\_\_, \_\_\_, \ldots\}$ $\{2, 3, 5, 7, 11, \_\_\_, \_\_\_, \_\_\_, \ldots\}$	See addenda.
3. Cross Product of Sets, Ordered Pairs	The student can form from two given sets the product set using ordered pairs.	Set A = $\{a, b\}$ Set B = $\{x, y, z\}$ $A \times B = \left\{\begin{matrix}(a,x), (a,y), (a,z)\\(b,x), (b,y), (b,z)\end{matrix}\right\}$
4. Relations	(a) The student can state and illustrate that a relation is a set of ordered pairs.	Example: $\{(1,3), (2,4), (3,5), (4,6)\}$

Major Topic	Behavioral Objectives	Teaching Aids
Functions	(b) The student can tell which of the following relations are functions. 1) $\{(1,2), (2,3), (3,4)\}$ 2) $\{(1,2), (2,3), (3,5)\}$ 3) $\{(1,1), (1,2), (1,3)\}$	Definition: A function is a relation (i.e., a set of ordered pairs) in which no two different ordered pairs have the same first element. For example: (2.3) (2,4) is not a function because the first elements, 2, are the same, but the second elements, 3 and 4, are not the same.
5. Graphing of Relations and Functions	The student can graph the following: 1) $x + y = 5$ 2) $x + y > 6$	In graphing an inequality, first graph the border, which in this case (b) is $x + y = 6$. Then, shade the graph on the side that contains points that make the inequality true.





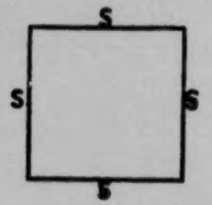
Major Topic	Behavioral Objectives	Teaching Aids
C. MATHEMATICAL SENTENCES		
1. Open, True and False Sentences	(a) The student can write, correct or finish mathematical sentences which are open, true or false.	See previous grades for examples.
	(b) The student can translate English sentences into mathematical sentences. For example: John has \$10.50. He bought a gift for his mother and then had \$4.30 left. How much did he spend for the gift?	$\$10.50 - x = \$4.30$
2. Equations	(a) The student can determine the solution set for simple equalities.	
Inequalities	(b) The student can determine the replacement set for simple inequalities.	$3 + 4 > \square$ Solution Set = $\{0,1,2,3,4,5,6\}$
3. Compound Conditions	The student can determine the solution set to mathematical sentences involving compound conditions. For example: $S < 7$ and $S > 3$.	If replacement set is the set of whole numbers, $S < 7$ and $S > 3$, then solution set would be $\{4, 5, 6\}$
4. Order of Operations	The student shall be able to use both brackets and parenthesis to show the order of operations in mathematical expressions involving addition, subtraction, multiplication and division.	$(5 + 3) \times 4 =$ ____ $(5 + 3) - [(4 - 3) \times 6] =$ ____

Major Topic	Behavioral Objectives	Teaching Aids
D. SYSTEMS AND LOGIC		
1. Probability	The student can answer questions such as the following: There are 9 black checkers and 15 red checkers in a box. What is the probability of selecting on the first draw:	Use checkers, coins, dice, and other small objects for experiments.
	1) a black checker?	1) 9/24
	2) a red checker?	2) 15/24

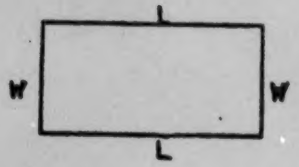
<table>
<tr><th>Major Topic</th><th>Behavioral Objectives</th><th>Teaching Aids</th></tr>
<tr><td>A. NON-METRIC</td><td></td><td></td></tr>
<tr><td>1. <u>Topology</u></td><td>Given a convex polyhedron, the student can use Euler's formula, namely, $V + F = E + 2$ where V = <i>vertices</i>, F = <i>faces</i>, and E is the <i>number of edges</i>.</td><td>The teacher should take solid figures, such as a <i>cube, tetrahedron, pyramid,</i> etc., and have students provide entries to a chart such as the following:
<table>
<tr><th>Object</th><th><i>Vertices</i></th><th><i>Faces</i></th><th><i>Edges</i></th></tr>
<tr><td>Cube</td><td>8</td><td>6</td><td>12</td></tr>
<tr><td>Tetrahedron</td><td>4</td><td>4</td><td>6</td></tr>
<tr><td>Etc.</td><td></td><td></td><td></td></tr>
</table>
On assembling the data, the student should be led to perceive the relationship between these qualities.</td></tr>
<tr><td>B. METRIC GEOMETRY</td><td></td><td></td></tr>
<tr><td>1. <u>Perimeter</u></td><td>(a) Given a plane or solid figure, the student will estimate the measure of its perimeter.</td><td>Students will wrap a string around the object and then measure the string using a standard measure.</td></tr>
<tr><td></td><td>(b) Given a plane or solid figure, the student will take a measure of its perimeter using a standard unit of measure (as ruler) and state its perimeter.</td><td>A trundle wheel device can be made to take measure of distances <i>in</i> or <i>out</i> of school building. Cut a wheel of whatever size is appropriate to the task, fix it to a pin or nail, wheel it around the border and count the turns. Multiply the number of revolutions times the circumference of the wheel to find the perimeter.</td></tr>
</table>

See addenda on MEASUREMENT.

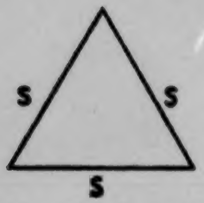
Major Topic	Behavioral Objectives	Teaching Aids
1. Perimeter (continued)	(c) Given certain geometric figures, the student will *select, utilize,* and *explain* the appropriate formula for the perimeter; for instance, the formula for a *square, rectangle,* and *triangle*: Example of perimeter: The distance an ant would walk if it started at one corner of a rectangle and kept walking along the side until it returned to its starting point. Applying the above *ant explanation* to the following pictures, lead students to develop for themselves the correct arithmetic sentence to fit each picture.	



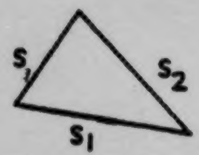
S+S+S+S = P or
P = 4S = P



L+W+L+W = P or
2L + 2W = P, P = 2 (L+W)



(equilateral)
S+S+S = P or
3S = P, P = 3S



(Scalene)
$S_1 + S_2 + S_3 = P$ or
$P = S_1 + S_2 + S_3$

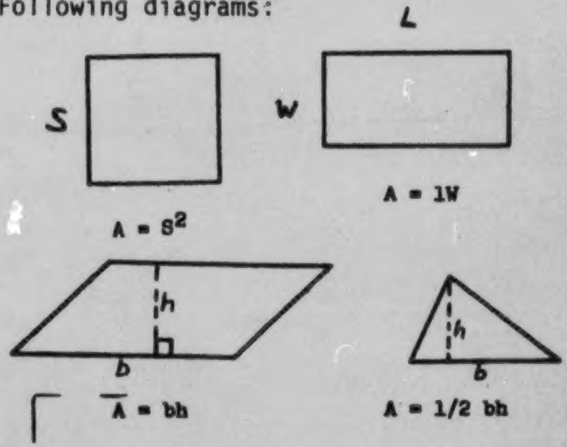
Major Topic	Behavioral Objectives	Teaching Aids
1. Perimeter - Angles (continued)	(d) Given various polygons cut from paper, the student will attempt to determine the sum of the angles.	Telling should be withheld, if possible. Some students will discover that any polygon may be divided into triangles. They can make charts of their findings.

Number of Sides	Sum of Angles
3	180
4	360
5	540
6	720
.	.
.	.
.	.

If regular shapes are used, students should be asked to determine the size of the angles occurring in the figure.

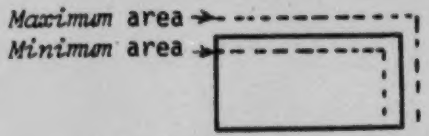
No. of Sides	Sum of Angles	Angle Measurement
3	180	60
4	360	90
5	540	108
6	720	120
.	.	.
.	.	.
.	.	.

Major Topic	Behavioral Objectives	Teaching Aids
2. Area	The student will compute the area for various objects utilizing and explaining the appropriate formula.	Following diagrams:


S
A = s^2
L
W
A = lW
h
b
A = bh
h
b
A = 1/2 bh

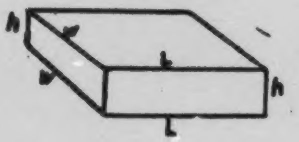
Greatest possible error may be defined as the difference between calculated maximum or minimum area and calculated measured area.

Possible Measurement

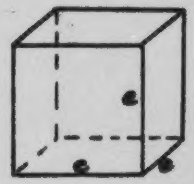

Maximum area
Minimum area

True area lies somewhere between the measured *maximum* and *minimum* areas.

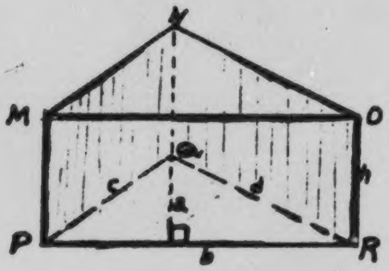
Major Topic	Behavioral Objectives	Teaching Aids
2. Area (continued)		Surface area -



$A = (lW + Wh + lh)$



$A = 6e^2$

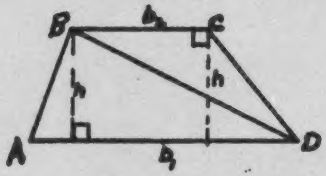
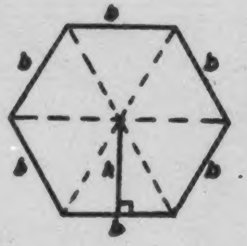


The area of △ MNO + area of △ PQR is the area, *ab*, of a parallelogram. The areas *hb*, *hc*, and *hd* are the areas of the three side faces of the prism.

The total surface area of the prism is represented by:

$$A = ab + hb + hc + hd$$

$$A = ab + h \cdot (b+c+d)$$

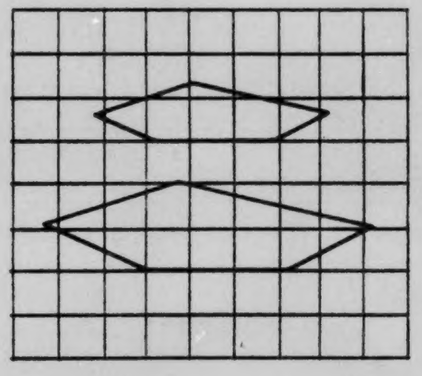
Major Topic	Behavioral Objectives	Teaching Aids
2. Area (continued)		Examples: Area of trapezoid  $A = \frac{1}{2}hb_1 + \frac{1}{2}hb_2$ Area of regular hexagon  *Hexagon* A = 1/2 hp ($p = 6b$) *Decagon* A = 1/2 hp ($p = 10b$)

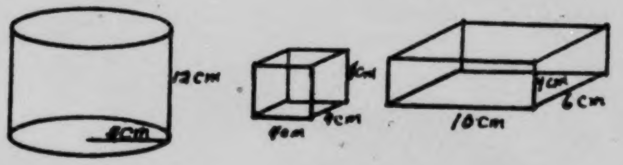
Major Topic	Behavioral Objectives	Teaching Aids
3. Volume	The student will *select, utilize,* and *explain* an acceptable way for finding the volume of figures such as prisms.	The intuitive ideas may be developed by the use of various sizes. 1. Study of the cube as a standard unit for measuring volume because it fills up one unit of space. 2. Student demonstrates ability to compare volume of right prisms making use of: a) Standard units of measure, *metric* *Metric*: cu. cm , cu. meter b) The mathematical sentences: 1) $V = e^3$ (cube) 2) $V = lwh$ (rectangular right prisms) 3. Student recognizes that: a) The idea of greatest possible error can be applied to volume of solids. b) *True volume* lies between the possible maximum and minimum measured volume.

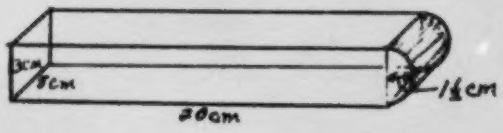
MEASUREMENT AND ESTIMATION GRADE SEVEN

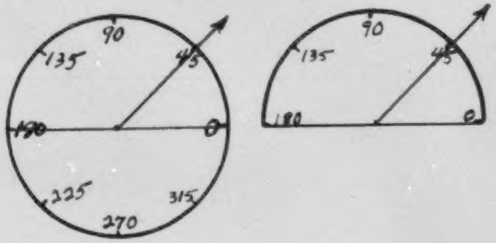
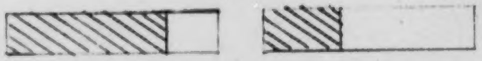
Major Topic	Behavioral Objectives	Teaching Aids
A. CONCEPT OF MEASUREMENT		
1. Process of Measuring	(a) Given a measurable physical property such as length, area, weight, temperature, etc., the student can *select* a suitable unit and/or measuring device and measure the property. For example: An angle can be measured by using a (n) _____. What is an appropriate unit when discussing the weight of diamonds?	Have the students attempt to describe in their own words what is involved in measurement. Use these three steps in measuring: 1. selecting a unit 2. dividing object into these units 3. counting the units
Arbitrary Selection of Units	(b) The student can name one unit suitable for naming the measure of a given physical property. For example: The weight of a box of cereal can be expressed in grams.	Cut pieces of string to lengths equal to certain parts of the body, such as foot, arm, hand. Use these pieces to measure the length of the room, blackboard, table, etc. Compare these by using standard units of measure. (centimeters or meters)
Approximate Nature of Measurement	(c) The student can demonstrate his understanding of the approximate nature of measurement by stating the precision of the measure. For example: The thickness of a piece of paper is .008 centimeters correct to the nearest thousandeth of a centimeter.	
2. Concept of Graphs	(a) The student can obtain information from different kinds of graphs (bar, circle, line, etc.) by solving computational problems using this information.	Find a collection of graphs large enough for classroom use. These may be graphs made by students.
	(b) The student is able to gather and tabulate information to be used in constructing graphs. For example: The population of the ten largest cities in Idaho. The average maximum temperature for the month of ____________.	Use of social studies and science textbooks for graphs and possible ideas for construction of graphs will be helpful.

Major Topic	Behavioral Objectives	Teaching Aids
B. MEASURABLE QUANTITIES		
1. Time	(a) The students can answer questions pertaining to time such as: How many decades of the 20th century have passed?	Give the students a chance to discuss the various ideas concerning time, decade, centuries, millenium, etc.
	(b) If a person is 11 years old, he is ______ days old, _____ hours old. If it is 9:00 o'clock on the West Coast, what time is it on the East Coast?	
2. Money Discount	The student can find the sale price of an item if he is given the original price and the discount.	Look for newspaper advertisements of sales.
3. Linear Measurement Metric Units	(a) Given a room, desk, or table to measure, the student can use various measuring devices (ruler, meterstick, tape, etc.) to measure length in whole and fractional parts of units.	Secure laboratory material. See addenda.
Perimeter	(b) The student can find the perimeter of polygons by either measuring the sides or if their measurements are given, by applying the appropriate formula.	Use geo-board, pegboard, grid paper, large cardboard shapes of polygons for the students to measure.
Circumference	(c) Given the radius or diameter of a circular object, the students can measure the circumference of a circular object, or he can calculate the circumference.	Find large cardboard circles for the students to measure; also jars, cans, and other objects with circular faces. A trundle wheel may be used. See addenda.

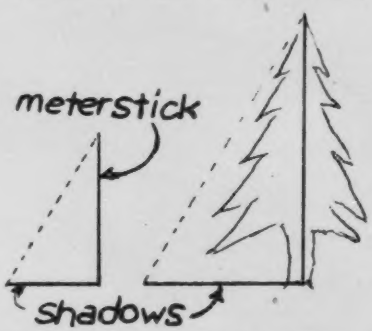
Major Topic	Behavioral Objectives	Teaching Aids
4. **Area** Circle Rectangle Triangle Parallelogram	(a) The student can demonstrate his ability to find the area of regular quadrilaterals, triangles and circles. (b) Given irregular figures sketched on grids, the student can find the approximate area of these figures by counting the number of unit squares and parts of squares contained in each.	Use geo-board, squared grid paper, tiles, etc. Use non-standard units of measure such as how many hands would it take to cover the top of the desk to help develop the idea of area. Let students compare similar shapes for areas by counting the squares and parts of squares in the interior of figures on grid paper.  See Grade III, Section B. 4. (a).
Surface Area Rectangular Prism Cube Cylinder	(c) Given prisms, cubes, or cylinders and measuring device, the student can find the surface area.	Get boxes and cans for the students to measure.



Major Topic	Behavioral Objectives	Teaching Aids
5. Volume Rectangular Prism Cube	(a) Given the appropriate measurements the students can find the volume of cylinders, cubes and rectangular solids.	Enrichment activity: The student can put the formulas for rectangular solids and cylinders together to find the volume of solid figure below. 
Liquid Measure	(b) The student can name the common standard units of liquid measure and measure the capacity of a given container to the nearest whole or fractional part.	Use laboratory material. See addenda.
6. Weight - Mass	Given a set of objects the student can name an appropriate unit of weight for measuring them.	Use laboratory material. See addenda.
7. Temperature Degrees Celius	Using a thermometer the student can read the temperature correct to the nearest degree.	Use different types of thermometers. Candy, out-door, in-door, etc.

Major Topic	Behavioral Objectives	Teaching Aids
8. Angle	The student can use the protractor to measure the angles of simple polygons.	Use both a circular and semicircular protractor for measuring angles.  90 135 45 180 0 225 270 315 90 135 45 180 0
9. Speed	The student can answer questions similar to the following pertaining to speed: The speed of sound at sea level is 1287 km/hr. Find this speed of sound to the nearest tenth of a unit in: km per minute, km per second, meters per second.	
10. Estimation	Given situations where measurement or counting can be used on a problem, the student can estimate the measurement or count within reason.* For example: Estimate what percent of each region is shaded. 	Exhibit an empty paper bag and a set of objects, all of which are the same size; blocks, chalk, erasers, etc. Have the student guess how many objects will fit into the paper bag. After guessing, fill bag to see who made the best guess.
11. Graphs Coordinate	(a) The student can graph the solution set to problems of the following type on a coordinate axis: $x + y = 5$; $x < 2y$	Reading road maps is one way of introducing coordinate points. Also use Longitude and Latitude on maps in an Atlas. Use pegboard, ruled graph paper, and the Battleship game. See addenda. Get a collection of different types of graphs. Play Tic-Tac-Toe. See addenda.

*"Within reason" is defined as within 10% error of the measurement or count.

Major Topic	Behavioral Objectives	Teaching Aids
Bar Line Circle	(b) From collected or given data the student can make graphs of the following types: Bar, line, circle, pictographs.	Use social studies and science texts for data, or collect own data.
12. Accuracy and Greatest Possible Error	The student can complete questions such as: _______ is the greatest possible difference between the real length of a segment and the measurement that is stated.	
13. Indirect Measurement Scale Drawings Similar Polygons	The student can use scale drawings to answer questions such as: If the shadow of the meter stick is 70 cm and the shadow of the tree is 560 cm, how tall is the tree?  	Let students make similar measurements of objects on the school ground; (flag pole, trees).
14. Statistics Mean (average) Mode Median	Given a set of data, 5,9,13,13,15,9,17,9 the student can answer the following questions: 1. The mode of the above set is ______. 2. The mean of the above set is ______. 3. The median of the above set is _____.	Have students collect their own data.

Major Topic	Behavioral Objectives	Teaching Aids
C. RENAMING MEASURES		
Comparison of Units Conversion of Units	The student can express the relationships between units of measure, appropriate to the grade level.	
D. COMPUTATIONS WITH MEASURES		
Conversions with Operations	The student can compute with measures appropriate to the grade level, assign the proper unit to the result, and rename if necessary. For example: A trip of 2880 kilometers was made in 8 hours. What is the rate in kilometers per minute?	

Major Topic		Behavioral Objectives	Teaching Aids
A. THE NATURAL NUMBERS - COUNTING AND NUMERATION			
1. Primes and Composite		Given any reasonable whole number, the student must be able to determine its complete factorization.	See Grade VII, Section A. 1. (a) - Teaching Aids.
2. Exponential Notation	(a)	The student must be able to write any reasonable number in expanded notation using exponents.	For example: 123,456,789 = $(1 \times 10^8) + (2 \times 10^7) + (3 \times 10^6) +$ $(4 \times 10^5) + (5 \times 10^4) + (6 \times 10^3) +$ $(7 \times 10^2) + (8 \times 10^1) + (9 \times 10^0)$ Note: $10^0 = 1$
	(b)	The student must be able to write any reasonable number in scientific notation.	$93{,}000{,}000 = 9.3 \times 10^7$
B. THE NATURAL NUMBERS OPERATIONS			
Greatest Common Factor Least Common Multiple	(a)	Given a set of three numbers having less than four digits, the student must be able to determine the greatest common factor.	See Grade VII, Section B. 1. (a) and (b) - Teaching Aids.
	(b)	Given a set of numbers having less than four digits, the student must be able to determine the least common multiple.	

Major Topic		Behavioral Objectives	Teaching Aids
C. THE RATIONAL NUMBERS			
1. Equivalent Fractions and Inequalities		Given a set of rational numbers, the student must be able to determine whether any two are equivalent or if they are not, they must determine which is less and which is greater.	See Grade VII, Section C. 1. (a) and 2. (a).
2. Ordering Fractions		Given a set of ten fractions, the student must be able to order them from least to greatest.	
3. Number Pairs, Ratio, and Proportion		Given a proportion of the type $\frac{2}{3} = \frac{x}{9}$, the student should be able to determine the value of the missing component.	
4. Decimal Fractions	(a)	Given a set of terminating decimal fractions, the student should be able to determine the equivalent common fractions.	
	(b)	Given a set of common fractions, the student should be able to determine the equivalent decimal fractions.	
D. THE RATIONAL NUMBERS OPERATIONS			
1. Addition and Subtraction of Fractions		Given the fractions of the type $\frac{37}{39}$ and $\frac{19}{48}$, student should be able to determine the sum and difference.	The student can use his ability to find the least common multiple to determine the lowest common demoninators. Multiplication Table - see addenda.

	Major Topic	Behavioral Objectives	Teaching Aids
2.	Properties of Addition and Subtraction	Given equations of the following type, the student should be able to solve them: $\frac{9}{16} + \left(\frac{3}{4} + \frac{5}{9}\right) = \Box + \frac{3}{4} \quad + \frac{5}{9}$ $\frac{17}{21} + \frac{0}{21} = \Box$; $\frac{3}{7} + \frac{6}{15} + \frac{6}{15} + \Box$	
3.	Multiplication and Division of Fractions	Given fractions of the type $\frac{16}{39}$ and $\frac{78}{25}$, the student should be able to determine the product and quotient.	The student can use his ability to determine the greatest common factor to simplify products and quotients.
4.	Properties of Multiplication and Division	Given equations of the following type, the student should be able to solve them: $3 \cdot 5 = 5 \cdot$ ____ $\frac{5}{7} \cdot \left(\frac{4}{7} \cdot \frac{5}{3}\right) = \frac{5}{7} \cdot \Box \cdot \frac{5}{3}$ $\frac{13}{27} \cdot \frac{27}{27} = \Box$ $\frac{15}{19} , \left(\Box\right) = \frac{15}{19}$	The student can use his ability to determine the greatest common factor to simplify products and quotients.

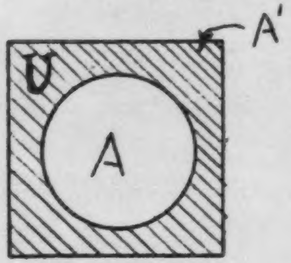
Major Topic	Behavioral Objectives	Teaching Aids
E. THE INTEGERS		
1. Properties of Naturals Extended to Integers	Given equations of the following type, the student should be able to solve them: $^{+}7 + ^{-}9 = ^{-}9 + \underline{\qquad}$ $^{-}3 \cdot (^{+}4) = ^{+}4 \cdot (\underline{\qquad})$ $^{+}9 + (^{-}3 + ^{-}5) = (^{+}9 + \underline{\qquad}) + ^{-}5$ $^{-}7 \cdot (\underline{\qquad} \cdot ^{-}3) = (^{-}7 \cdot ^{+}4) \cdot ^{-}3$ $^{-}3 \cdot (^{+}4 + ^{-}7) = ^{-}3 \cdot (^{+}4) + ^{-}3 \cdot (\underline{\qquad})$ $^{-}5 + \underline{\qquad} = ^{-}5$ $^{-}3 \cdot (\underline{\qquad}) = ^{-}3$	
2. Absolute Value	Given a set of integers the student must be able to write the absolute value of each integer.	$\lvert ^{+}3 \rvert = 3$ $\lvert ^{-}3 \rvert = 3$ $\lvert 0 \rvert = 0$
3. Addition and Subtraction of Integers	Given problems of the following type, the student should be able to solve for the variable. $^{+}27 + ^{-}43 = X$ $^{-}93 - ^{-}47 = Z$	Refer: Grade VII, Section E. 2. (a).

Major Topic	Behavioral Objectives	Teaching Aids
4. Multiplication and Division of Integers	Given problems of the following type, the student should be able to solve for the variable. $^{-}12 \cdot (^{+}7) = X$ $144 - (^{-}16) = Z$	$\sqrt{32} = 4\sqrt{2}$
F. THE IRRATIONAL NUMBERS		
1. Simplification and Operations with Radicals	(a) Given a set of radicals, the student must be able to simplify each.	$\sqrt{\frac{3}{4}} = \frac{\sqrt{3}}{\sqrt{4}} = \frac{\sqrt{3}}{2}$ $\sqrt{\frac{4}{3}} = \frac{\sqrt{4}}{\sqrt{3}} = \frac{2}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$
	(b) Given a set of radicals, the student must be able to perform each of the basic operations with the numbers.	$\sqrt{2} + 3\sqrt{2} = 4\sqrt{2}$ $7\sqrt{3} - 2\sqrt{3} = 5\sqrt{3}$ $\sqrt{2} \cdot \sqrt{3} = \sqrt{6}$ $\frac{\sqrt{12}}{\sqrt{3}} = \sqrt{\frac{12}{3}} = \sqrt{4} = \pm 2$ because $\sqrt{x^2} = \lvert x \rvert$ by definition

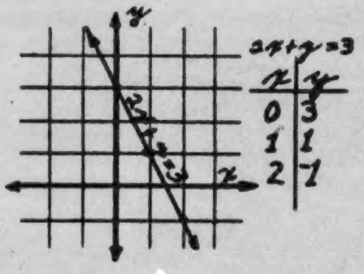
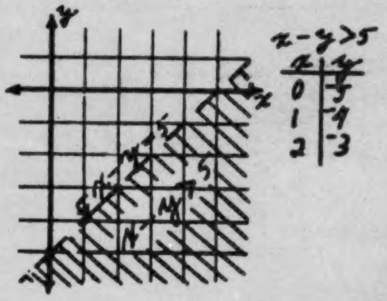
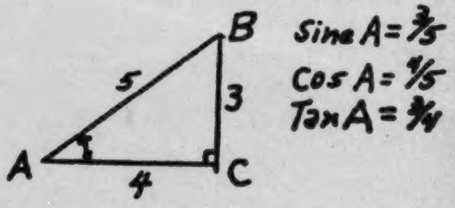
SETS, FUNCTIONS, RELATIONS, SYSTEMS, AND LOGIC

GRADE EIGHT

Major Topic	Behavioral Objectives	Teaching Aids
A. SETS		
1. Collections	(a) Given a description of a set, the student can list the numbers of the set using the roster form.	Set C contains all whole numbers between 17 and 21 Set C = $\{18, 19, 20\}$
	(b) Given a set in roster form, the student can give a description of the set.	Set D = $\{1, 4, 9, 16\}$
	(c) The student can state the definitions of geometric figures as the union of sets of points.	Set D contains the squares of the whole numbers between 0 and 5.
Elements	(d) Given a description of a set, the student can tell whether it is an element or is not an element of the set.	For example: "A segment is the set of points whose members are two given points and all points between these two points."
2. Equivalent and Non-equivalent Equal and Non-equal Sets	(a) Given two or more sets, the student can compare the sets and tell whether they are equivalent or non-equivalent, equal or non-equal.	
Subsets Proper and Improper	(b) Given a set, the student can form the proper and improper subsets of it.	
3. Cardinal Number 0 - Infinity	The student can determine the cardinal number of a set.	
4. Commutative, Associative and Distributive Property	The student can use sets to demonstrate the commutative, associative and distributive properties.	See previous grades for illustrations.

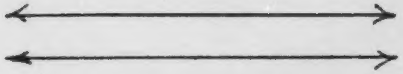
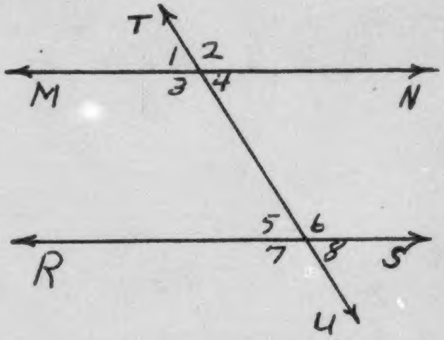
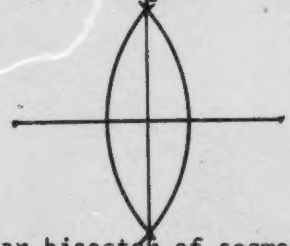
Major Topic	Behavioral Objectives	Teaching Aids
5. Union and Intersection of Sets	Given two sets, the student can form the union and the intersection of the sets.	
6. Universal Set Complement	(a) Given a set, the student can state an appropriate universal set.	Set of math books, the Universal set is the set of all books.
	(b) Given a set and a universal set, the student can give the complement of the given set.	 Complement of A is the shaded area.
7. Finite, Infinite and Empty, Non-empty	Given a list of sets, the student can classify them as finite or infinite, empty or non-empty.	
8. Venn Diagrams	The student can use Venn diagrams to show the union and intersection of two or more sets.	See Grade VII, Section 7. A. (a)
9. Set Symbolism	The student can use the following set notation correctly.	For example: $A = \{a, b, c\}$; $B = \{b, c\}$, then: $B \subset A$; $n(A) = 3$; $A \cap B = \{b, c\}$
B. FUNCTIONS AND RELATIONS		
1. "What's My Rule?"	Given the game situation described in the addenda, the student shall be able to determine the missing values, and state the rule of the game.	See addenda. See also previous grades for examples.

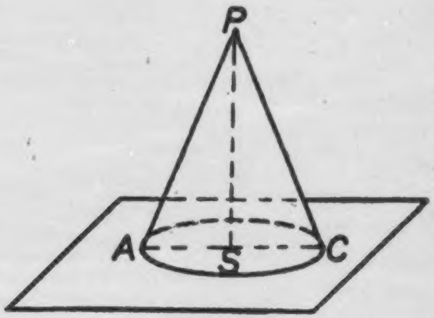
Major Topic	Behavioral Objectives	Teaching Aids
2. Number Patterns	The student shall be able to complete the next few values in the number patterns. They will also be able to describe the pattern in terms of a mathematical sentence. $\{1, 2, 3, 5, 8, \_\_\_, \_\_\_, \_\_\_, \ldots\}$ $\{18, 15, 12, 9, \_\_\_, \_\_\_, \_\_\_, \ldots\}$	See addenda.
3. Cross Product of Sets, Ordered Pairs	The student can form from two given sets the product set using ordered pairs.	Set A = $\{1, 2, 3\}$ Set B = $\{4, 5, 6, 7\}$ A X B = $\{(1,4), (1,5), (1,6), (1,7), (2,4), (2,5), (2,6), (2,7), (3,4), (3,5), (3,6), (3,7)\}$
4. Relations	(a) The student understands that a relation is a set of ordered pairs.	Example: $\{(1,3), (2,4), (3,5), (4,6)\}$
Functions	(b) The student can answer correctly the following question: Which of the following relations are functions? 1) $\{(1,2), (2,4), (3,6), (4,8), (5,10)\}$ 2) $\{(1,3), (2,3), (3,3), (4,3)\}$ 3) $\{(3,1), (3,2), (3,3), (3,4)\}$ 4) $\{(1,1), (1,2), (2,2)\}$	See Grade VII, Section B. 4. (a).

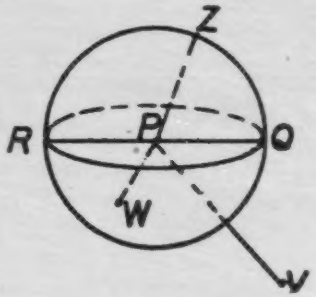
Major Topic	Behavioral Objectives	Teaching Aids
5. Graphing of Relations and Functions	(a) The student can graph the following: 1) $2x + y = 3$ 2) $x - y > 5$	 
Trigonometric Functions	(b) Given a right triangle, the student can find the sine, cosine and tangent in terms of the sides.	 A B C 5 3 4 Sine A = 3/5 Cos A = 4/5 Tan A = 3/4

Major Topic	Behavioral Objectives	Teaching Aids
C. MATHEMATICAL SENTENCES		
1. Open, True and False Sentences	(a) The student can write mathematical sentences which are open, true or false.	See Grade VII, Relations and Functions 5. (a).
	(b) The student can translate English sentences into mathematical sentences. For example: John has \$10.50. He bought a gift for his mother and then had \$4.72 left. How much did he spend for the gift?	$\$10.50 - x = \$4.72$
2. Equations - Inequalities	From a given set, the student can select the replacement set for simple equalities and inequalities.	Set of whole numbers as the replacement set. $x + 4 = 25$ $3x - 2 < 10$ Solution Set $\{21\}$ Solution Set $\{1, 2, 3\}$
3. Compound Conditions	From a given replacement set, the student can determine the solution set for mathematical sentences involving compound conditions.	Replacement set is the set of whole numbers. $x + 2 > 8$ and $x - 4 < 9$ Solution Set $\{7, 8, 9, 10, 11, 12\}$
4. Order of Operations	The student shall be able to use both brackets and parentheses to show the order of operations in mathematical expressions involving addition, subtraction, multiplication, and division.	$3\left[(5-2) \times (4-2)\right] = \_\_\_\_$
5. Absolute Value	The student can find the solution set to solute value.	$\lvert y \rvert + 2 \leq 5$

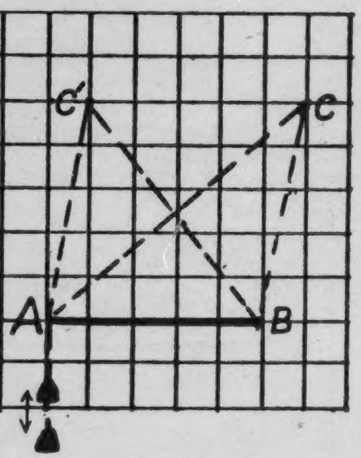
Major Topic	Behavioral Objectives	Teaching Aids
D. SYSTEMS AND LOGIC		
1. Probability	The student can answer questions such as the following: A bag contains 5 white marbles, 3 green marbles, and 7 red marbles. What is the probability of selecting on the first draw:	Marbles, checkers, dice, and other small objects can be used for experiments.
	1) a white marble?	1) 1/3
	2) a green marble?	2) 1/5
	3) a red marble?	3) 7/15
	4) a white or green marble?	4) 8/15
2. If and Only If	The student can complete the following:	
	A conjunction is __________ if and only	True
	if both simple sentences are __________.	True

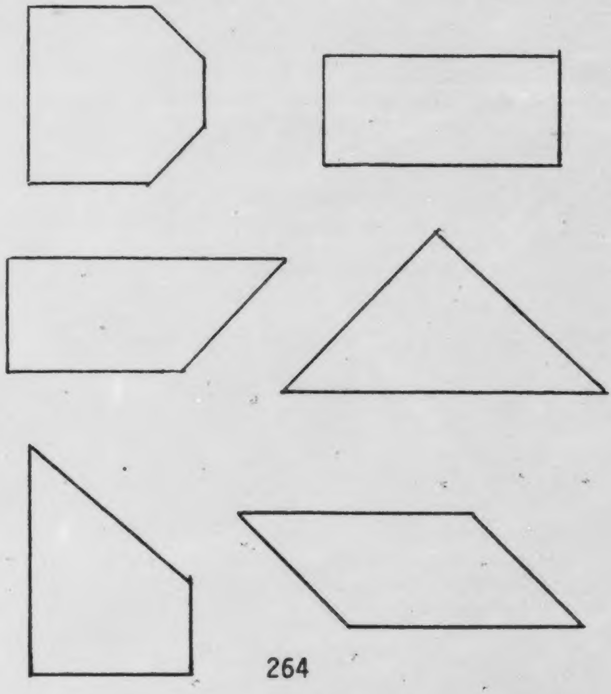
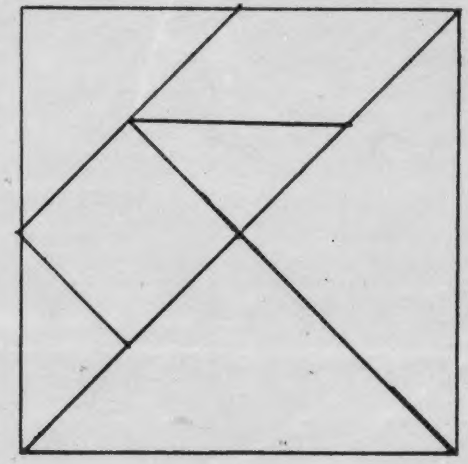
Major Topic	Behavioral Objectives	Teaching Aids
A. NON-METRIC		
1. Parallel Lines	(a) The student will *recognize, draw, label,* and *symbolize* parallel lines. Given two lines on paper or on the chalkboard, he can show whether the lines are parallel and *explain* this finding. 	The student could be required to point out in the classroom examples of parallel lines; i.e., the opposite sides of a table, the lines made by the wall meeting the ceiling and floor, etc.  T 1 2 M 3 4 N 5 6 R 7 8 S U
Transversals and Parallel Lines	(b) The student shall *recognize, draw,* and *label* the corresponding, alternate interior angles and alternate exterior angles.	If MN is parallel to RS, then angles 4 and 8, 6 and 2, 1 and 5, 3 and 7, are examples of pairs of corresponding angles. Angles 3 and 6, 4 and 5 are pairs of alternate *interior* angles. Angles 1 and 8, 2 and 7 are pairs of alternate *exterior* angles.
2. Perpendicular	(a) Given a line, the student can construct a line perpendicular to it.	 Perpendicular bisector of segment.

Major Topic	Behavioral Objectives	Teaching Aids
2. Perpendicular (continued)	(b) Given a line and an intersecting line, the student can *demonstrate* whether they are perpendicular.	The student can check with the aid of a compass or protractor; or, he can fold paper to create a model of a right angle and apply it.
3. Cones	The student will *identify*, *name* and *describe* the parts of a right circular cone.	See drawing below:  The student should be able to name AC as the diameter, SP as the height of the cone. AP is the slant height. Point P is the vertex. The student should have some idea of the lateral surface. If the teacher will make some paper cones and have the student unroll them, he would gain some insight into lateral surface area.

Major Topic	Behavioral Objectives	Teaching Aids
4. Cylinder	(a) The student will *recognize, identify,* and *name* parts of a right circular cylinder.	The student should be able to name cylinders with which he is familiar. He should be able to name the base, diameter, lateral surface of a cylinder.
	(b) Given a model of a right circular cylinder, the student will be able to check to see if it meets the criteria for a cylinder.	The student should check for congruent bases, the perpendicularity of the surface to the bases and the fact that the bases are circular.
5. Sphere	(a) The student can *recognize, identify* and *name* the parts associated with a sphere.	A sphere is a set of all points in three-space that are the same distance from some given point.
	(b) Given a piece of string and a point, such as the end of a pencil, the student can *describe* the sphere which could be made by using these materials.	The following diagram illustrates that point P is the *center*; segments PZ, PR, and PQ are *radii*. Segment RPQ is a *diameter*.  Referring to the above figure, we note the identifications on the following page.

Major Topic	Behavioral Objectives	Teaching Aids
5. Sphere (continued)		Behavioral Objective (b) continued: 1. All points *X* (any arbitrary point) so that $\overline{PX} < \overline{PQ}$ or X=P are *interior points*. (Since $\overline{PW} < \overline{PQ}$, *W* is an *interior point*.) 2. All points *X* so that $\overline{PX}=\overline{PQ}$ are *in* (on) the sphere itself. (Thus, Z is *in* the sphere since $\overline{PZ} = \overline{PQ}$.) 3. All points *X* such that $\overline{PX} > \overline{PQ}$ are *exterior points*. (Since $\overline{PV} > \overline{PQ}$, *V* is an *exterior point*.)
B. METRIC GEOMETRY		
1. Perimeter and Area	(a) On the geo-board, the student can *demonstrate figures* with various perimeters but with equivalent areas.	On the geo-board, the teacher could start with a small square and call it *one* and then ask the students to put a rubber band around as many shapes as possible that have an area of *two*.

Major Topic	Behavioral Objectives	Teaching Aids
1. Perimeter and Area (continued)		Refer: Behavioral Objective for (a). The pegboard could be used to an advantage to show some geometric figures can have a change in perimeter without a change in area.  Tie string to peg at A, then pass string around peg at B and over peg at C and down to A. Place weight on end of string to hold it tight. Move peg at C to various holes as C', keeping altitude constant but noting change in perimeter as weight goes up and down.

Major Topic	Behavioral Objectives	Teaching Aids
1. Perimeter and Area (continued)	(b) Using a string with a fixed perimeter, the student will arrange it to enclose different areas.	Use pegboard and change a rectangle to a parallelogram without changing the length of the sides. If the height of the rectangle is five, it can be readily changed to a parallelogram with a height of four and three without changing the length of the sides.
	(c) Using tangrams, the student will arrange the pieces in various patterns revealing different perimeters while the area remains constant. Some of the shapes that can be made are: 	Tangram pieces can be made from a square piece of cardboard using the following pattern:  Refer to TANGRAMS - 300 PUZZLES by Ronald C. Read

Major Topic

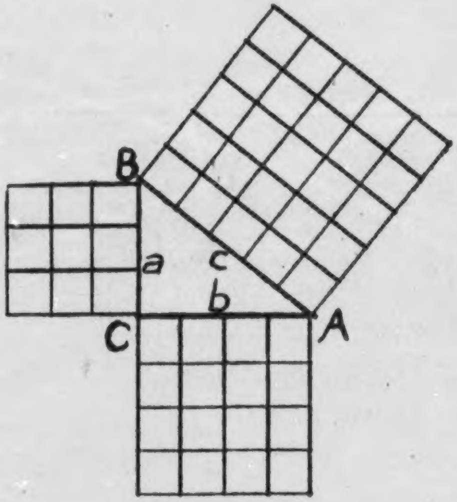
2. Pythagorean Property and Special Right Triangles

Behavioral Objectives

(a) Given a right triangle, for which the lengths of two sides are known, the student can compute the length of the third side.

(b) Given a problem which involves utilization of the *Pythagorean Property*, the student can demonstrate to the teacher or class the solution process.

Teaching Aids



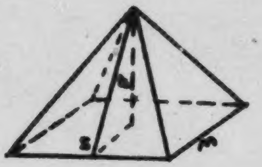
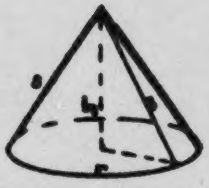
Pythagorean Property:

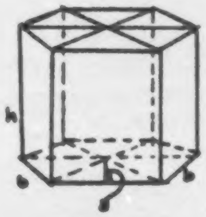
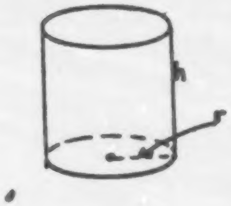
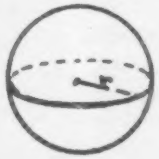
The area of the square on AB equals the sum of the areas of the squares on BC and AC. AB = c = 5; AC = b = 3; and BC = a = 4.

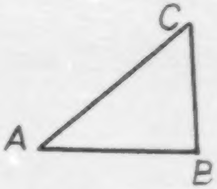
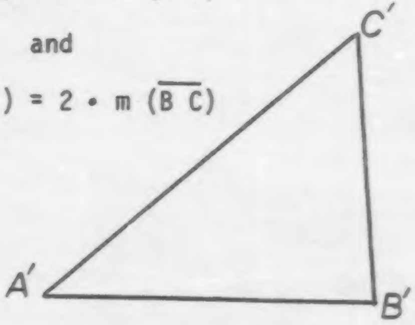
$$c^2 = a^2 + b^2$$

For example:

The student working with other materials (chalkboard, paper, geo-board) can diagram and explain the problem.

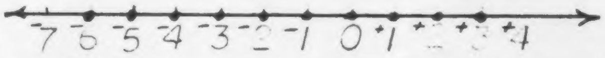
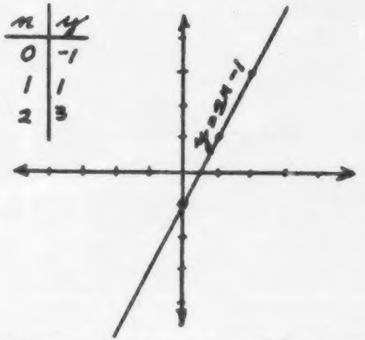
Major Topic	Behavioral Objectives	Teaching Aids
3. Volume	The student will *select, utilize,* and *explain* an acceptable way for finding the volume of figures such as *hexagonal prisms, cylinders, pyramids,* and *cones.*	 Square Pyramid V = 1/3 Bh Lateral Surface = 1/2 ps (not base) (p = 4n) (s = alt. each face)  Right Circular Cone V = 1/3 Bh *or* $V = 1/3\ r^2h$ Lateral Surface A = 1/2 cs (does not include base) (c = circumference of base) (s = slant height)

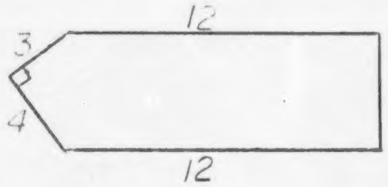
Major Topic	Behavioral Objectives	Teaching Aids
3. Volume (continued)		 NOTE: B = area of base S = surface Hexagonal Right Prism V = Bh *or* V = 1/2 aph (p = 6b) S = p (a+h)  Right Cylinder $V = Bh$ *or* $V = \quad r^2h$ $S = 2 \quad rh+2 \quad r^2$ $= 2 \quad r(h+r)$ Sphere $V = 4/3 \quad r^3$ $S = 4 \quad r^2$ 

Major Topic	Behavioral Objectives	Teaching Aids
4. Similarity and Proportions	(a) Given *similar* and *dissimilar* figures, student can *identify* those which are *similar* and demonstrate the methods of confirming that two triangles are similar.	Student learns the properties of measurement related to similar triangles. Definition: Two triangles are said to be similar if there is a one-to-one correspondence between the vertices so that corresponding angles are congruent and the ration of the measures of corresponding sides are equal, thus making corresponding sides proportional in length.  $\angle A \cong \angle A'$ $m(\overline{A'B'}) = 2 \cdot m(\overline{AB})$ $\angle B \cong \angle B'$ $\angle C \cong \angle C'$ $m(\overline{A'C'}) = 2 \cdot m(\overline{AC})$ and $m(\overline{B'C'}) = 2 \cdot m(\overline{BC})$ 

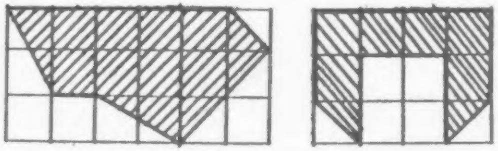
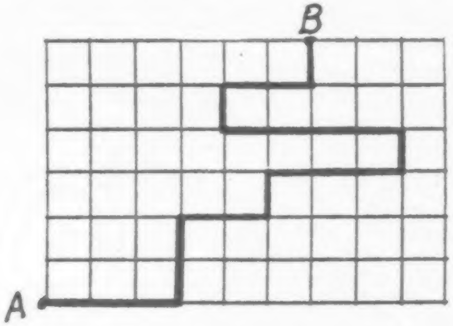
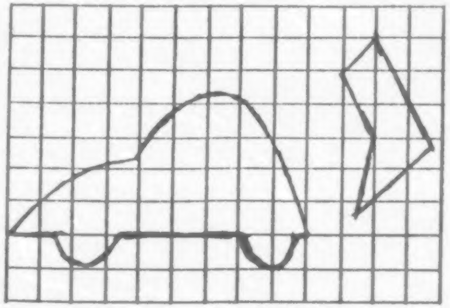
Major Topic	Behavioral Objectives	Teaching Aids
4. Similarity and Proportions	(b) Student demonstrates ability to measure height of tree, building, or pole, indirectly, by use of the properties of similar triangles.	A 240 cm shadow is cast by a man who is 180 cm tall. At the same time, a building casts a shadow 1200 cm long. How high is the building? Refer to GEOMETRY, Grade Seven, Section B-4 (a) and MEASUREMENT AND ESTIMATION, Grade Seven, Section B-4 (a).

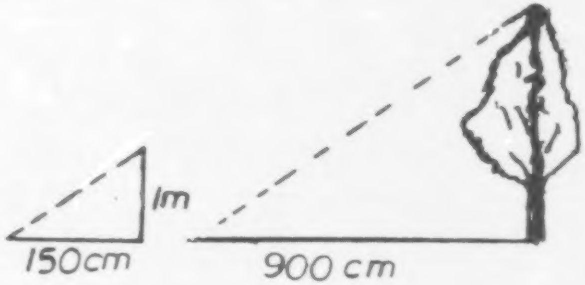
Major Topic	Behavioral Objectives	Teaching Aids
A. CONCEPT OF MEASUREMENT		
1. Process of Measuring	(a) Given a measurable physical property such as length, area, weight, temperature, etc., the student can select a suitable unit and/or measuring device and measure the property. For example: An angle can be measured by using a (n)_____. What is an appropriate unit when discussing the weight of diamonds?	Have students attempt to describe in their own words what is involved in measurement. Three steps used in measuring are: 1. selecting a unit 2. dividing the object into these units 3. counting the units
Arbitrary selection of unit	(b) The student can name at least two units suitable for naming the measure of a given physical property. For example: Speed can be expressed in kilometers per hour or meters per second.	Cut pieces of string to length equal to certain parts of the body. Use these pieces to measure length of room, blackboard, etc. Compare results with those obtained by using standard units of measure. Have a discussion on some of the early units of measurement; cubit, span, etc., and why there was a need for a standard unit of measurement.
Approximate nature of measurement	(c) The student can demonstrate his understanding of the approximate nature of measurement by stating the precision of the measure. For example: The thickness of a piece of paper is .008 centimeters, correct to the nearest thousandth of a centimeter.	Use laboratory material. See addenda.

Major Topic	Behavioral Objectives	Teaching Aids
2. Concept of graphs	The student can graph integers and integer pairs. For example: Draw a number line and graph the integers greater than -7 and less than 4. $-7 < n < 4$ where n is an integer  Graph the function: $y = 2n - 1$.  n: 0, 1, 2 y: -1, 1, 3	Use number line, and blackboard graph for demonstration. Have students plot their solutions on the number line or coordinate axis.
B. MEASURABLE QUANTITIES 1. Time	The student can answer questions pertaining to time such as: There are _____ seconds in 3 minutes. If it is 10 o'clock in Chicago, it is _____ o'clock in Seattle.	
2. Money Discount Interest Simple Compound	(a) The student can find the sale price of an item, if he is given the original price and the rate of discount. (b) Given the amount, rate and time, the student can find both the simple interest and compound interest. He also can compare the results and explain which is greater.	See newspaper advertisement of sales.

	Major Topic		Behavioral Objectives	Teaching Aids
3.	Linear Measurement Metric units	(a)	Given a room, yard, etc. to measure, the student can use various measuring devices (ruler, meterstick, tape, etc.) to measure length in whole and fractional parts of units.	Use laboratory materials. See addenda.
	Perimeter Rectangles Parallelograms Trapezoids Triangles Squares	(b)	The student can find the perimeter of polygons by either measuring the sides or, if their measurements are given, by applying the appropriate formula.	Use geo-board, pegboard, grid paper, large cardboard shapes of polygons for the students to measure.
	Circumference Circle	(c)	Given a large circular object, the student can measure the circumference of the object and/or calculate the circumference if he knows the radius or diameter.	Find large cardboard circles for the student to measure. Jars, cans, and other circular objects can be used. Use a trundle wheel. See addenda.
4.	Area Rectangles Parallelograms Triangles Trapezoids Squares Circles	(a)	The student can demonstrate his ability to find the area of quadrilaterals, triangles, circles.	Use geo-board, pegboard, grid paper, tiles, etc.
		(b)	Given an irregular figure drawn on graph paper, the student can find the approximate area of the irregular figure by counting the number of unit squares contained in each and/or by dividing it it up into shapes of figures that they can compute the area.	
	Surface Area	(c)	The student can find the surface area of rectangular prisms, cubes, spheres, pyramids, and cylinders if given the necessary measurements.	Use models of different geometric solids: boxes, cans, etc.

Major Topic	Behavioral Objectives	Teaching Aids
5. Volume Rectangular Solid Cylinders Cubes Spheres Cones	(a) Given the appropriate measurements, the student can find the volume of cylinders, cubes and rectangular solids.	Use laboratory material. See addenda.
Liquid Measure	(b) The student can name the common standard units of liquid measure and measure the capacity of a given container to the nearest whole or fractional part of the unit.	Use laboratory material. See addenda.
6. Weight - Mass	Given a set of objects, the student can name an appropriate unit of weight for measuring them.	Use laboratory material. See addenda.
7. Temperature	The student can relate to celsius -- normal body temperature, comfortable room temperature, etc.	
8. Angle Complementary Supplementary	(a) The student can use a protractor to measure an angle to the nearest degree.	Use both circular and semicircular protractors.
	(b) The student can solve problems involving the measure of angles in both degrees and minutes. For example: The complement of an angle of 45° 15' is____.	Make large cardboard figures containin angles for the students to measure.

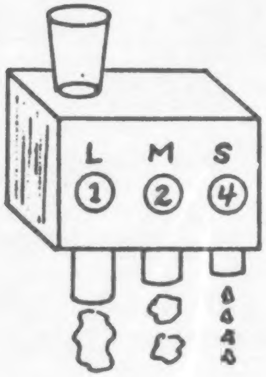
Major Topic	Behavioral Objectives	Teaching Aids
9. Speed	The student can answer questions similar to the following pertaining to speed: If an airplane travels 620 kilometers in one hour and would travel 580 kilometers per hour if there were no wind, is there a head wind or a tail wind? How fast is the wind?	
10. Estimation	The student can answer questions such as: Estimate the area of each of the following figures.  How far would one travel in going from A to B? 	Provied students with sets of cardboard squares and squares cut across one diagonal to form two triangles. Direct them to lay the squares and triangles out in various patterns and then find the area of the figure formed. Make figures similar to the following for the students to estimate the area of each figure.  See Grade Five, Section B-5 (a).

Major Topic	Behavioral Objectives	Teaching Aids
11. Graphs Bar, Broken line Circle	(a) From given data the student can select an appropriate graph and plot the data.	
	(b) The student can locate points on the coordinate axis.	See addenda for Battleship and Tic-Tac-Toe.
	(c) The student can plot the graph of the function $y = 2n - 1$.	See Grade Eight, Section A-2 (a).
12. Accuracy and Precision in Measurement	(a) The student can complete questions such as: Choose the measurement which is more precise -- 5×10^3 km. and 5.2×10^3 km.	
Greatest possible error	(b) The student can give the precision and greatest possible error for each of the following: 7.00 cm; 52.00 meters	
13. Indirect Measurement Scale Drawings Similar Polygons	The student can use scale drawings to answer questions such as: If the shadow of the meterstick is 150 cm and the shadow of the tree is 900 cm, how tall is the tree?  	Let students make similar measurements of objects on the school grounds. (Flag pole, trees).

Major Topic	Behavioral Objectives	Teaching Aids
14. Statistics Mean Mode Median Frequency distribution Histogram	The student can answer questions such as the following: Given the following set of data, scores on a math quiz were 78, 38, 75, 62, 65 53, 73, 73, 68, 56, 73, 70, 80, 39, 68, 75, 58, 78, 70, 60, 51 -- *i.* What was the highest score? The lowest score? *ii.* What is the mode for this set of data? *iii.* What is the median for this set of data? *iv.* What is the mean for this set of data? *v.* Construct a frequency distribution table and a histogram for this set of data.	Have the students collect their own data.
C. RENAMING MEASURES		
1. Comparison of Units Conversion of Units	The student can express the relationships between units of measure appropriate to the grade level and can rename a measure in other units. For example: 1 kilometer = _______ meters?	
D. COMPUTATIONS WITH MEASURES	The student can compute with measures appropriate to the grade level, assign the proper unit to the result.	

ADDENDA

R & R PACKAGING MACHINE



When looking for some way to introduce and enforce the concepts of place value, you might take a look at the Packaging Machine.

Upon examining the machine you will find it very versatile, easily adapted to your classroom situation and grade level. The machine itself could be a concrete model or an abstract drawing.

To introduce the machine to the pupils, the teacher could start with a story. Example:

I have a friend who invents machines. He is so busy inventing machines that he doesn't have a chance to see if they will work or not. To find this out, he has thought of a scheme which will let him keep on inventing and will furnish a way to test the machine. This is his scheme. He sends the plans to a friend and the friend tests the machine for him. I have just received a plan from him and would like you to help me test the machine.

(At this point the teacher could present a model or draw the machine on the board. An explanation of the machine could then be given by the teacher using this information.)

The machine is a packaging machine. The items that are being packaged (these could be almost anything and preferably suggested by the pupil) go in through the top and are automatically packaged in single packages, packages of ten, packages of one hundred, etc.

(Naturally, the size of the machine would be built to fit the particular classroom situation.)

The packages are then dropped through the tubes at the bottom of the machine. The machine always packages the largest packages first, next largest second . . . and the remaining items in single packages last. The round windows are called counting windows and are used to keep a record of the number of packages of various sizes made up by the machine for a particular order. This allows the operator to shut off the machine when an order is filled. It also gives him the information he needs to keep a record sheet which comes with the machine.

This is the record sheet that can be used with the machine.

R & R PACKAGE MACHINE RECORD SHEET

	ORDER	NUMBER OF BAGS IN EACH			TOTAL	
		LARGE	MIDDLE	SMALL	ITEMS	
		0	0	0	0	
1.	Jones	0	0	3	3	(0 x 100) + (0 x 10) + (3 x 1)
2.	Smith	0	0	9	9	(0 x 100) + (0 x 10) + (9 x 1)
3.	Peters	0	1	0	10	(0 x 100) + (1 x 10) + (0 x 1)
4.	Phillips	0	2	2	22	(0 x 100) + (2 x 10) + (2 x 1)
5.	Brown	1	2	4	124	(1 x 100) + (2 x 10) + (4 x 1)
6.						
7.						
8.						

At this point, the teacher could continue with the story of the inventor friend, stating that the friend would like a complete test made on the machine. Therefore, we should try the machine not only to see if it packages properly, but also to keep all the records just as though it was being used in a large factory.

The teacher could ask,

What numerals would appear in the counting windows before any items had been packaged?

These, of course, would be _zero_.

The numerals would then be recorded in the record sheet in their proper places with the proper total.

Next, three items could be put in the machine.

Use pictures or dots to represent these three items.

(This procedure should probably be continued through the introduction of smaller numbers and continued until it becomes unfeasible.)

Now the teacher could expand on the story about the inventor friend. It could be said that he asked that the machine be further checked to make absolutely sure it was accurate. Using expanded notation would be a good method for making this check.

Incidentally, if the zeros that precede the three in the package column become a problem in the total column, we might suggest that an odometer, which is familiar to most children, be used to straighten out the situation.

We would also suggest that all the zeros in the package counting windows be inserted in the package columns to avoid a misunderstanding later on in the expanded notation of (1 x 0) and (0 x 0).

A numeral with two like digits, such as twenty-two, could be used next to show that the two in the tens place is different than a two in the units place, etc. When the different steps used to operate the machine and record the numerals are mastered by the pupils, instead of putting items in the machine, the teacher could say,

> "What if a three appeared in the large window, a five in the middle window, and a two in the smallest window? What would appear in the record?"

To apply this machine to other bases as a means of strengthening the concept of base ten place value, the teacher could use the story approach.

> We could take the machine to Mars where people count differently than we do. We would suggest that another machine and record sheet, to be used with the new base, be drawn on the board next to the base ten machine as a reference for points that need to be made with the other base machine. The record sheet should probably have a total for bases other than ten as well as a base ten total. Otherwise, the same procedure would be used as was used with the base ten concepts.

THE BOARD *STRETCHERS*

The board *stretchers* are a set of *imaginary machines* which can be used to *stretch* boards. Thus, if a 4-decimeter board is put through a *3-stretcher,* it comes out 12 decimeters long. That is, 4 x 3 = 12. The children quickly see that the *stretchers* are merely a *set of multipliers.* To make the story a bit more plausible, it should be mentioned that once a board has been *stretched,* it becomes so tough and hard that it cannot be cut with a saw. Therefore, we need some board *shrinkers* as well as *stretchers.* Suppose a customer has a 6-decimeter board and wants it made into a board two decimeters long. What might he try? When this question is posed, invariably some child will suggest running the 6-decimeter length *backward through the 3-stretcher.* This, of course, is just what we want the children to see -- namely, that *division* is the *inverse operation* of *multiplication.* Thus, with this new operation, not only can we *stretch* boards, but we can also *shrink* them. There are two possible ways to interpret notation, either of which is appropriate.

4 x 3 = 12 may be interpreted as putting a 4-decimeter board through the *3-stretcher* giving a 12-decimeter length.

or

4 x 3 = 12 may be interpreted as implying that if a board is first run through a *4-stretcher,* then through the *3-stretcher,* it will come out 12 times as long.

The latter is probably more satisfactory for multiplying fractions.

Notation must also be developed for the *shrinking* operation.

If 3 denotes a *3-stretcher*, then we will say that $\frac{*}{3}$ denotes a *3-shrinker*.

The genius of this notation becomes apparent in describing an operation involving the use of a *2-stretcher* along with a *3-shrinker*, the notation for which is $\frac{2}{3}$.

This, of course, is precisely what the fraction $\frac{2}{3}$ does when *multiplied* by a number. It *stretches* it 2 times and *shrinks* it 3 times.

For example: $12 \times \frac{2}{3}$ can be interpreted as running a 12-decimeter through a *2-stretcher* and then through a *3-shrinker* to give an 8-decimeter length, so

$$12 \times \frac{2}{3} = 8$$

The other interpretation of

$$12 \times \frac{2}{3} = 8$$

is

if a board is first run through the *12-stretcher*, then through a *2-stretcher*, and finally through the *3-shrinker*, the result is the same as if the board had only been run through an *8-stretcher*.

In a similar manner,

$$\frac{3}{4} \times \frac{4}{9} = \frac{1}{3}$$

can be interpreted by noting if a board is successively run through a *3-stretcher*, a *4-shrinker*, a *4-stretcher*, and a *9-shrinker* the same thing is accomplished as if the board had simply been run through a *3-shrinker* in the first place.

Children can be given the opportunity to decide whether the order of operation makes any difference . . . . that is, does it matter whether we *stretch* first, then *shrink;* or *shrink* first, then *stretch.*

An interesting variation of the board *stretchers* is to develop a separate set of board *shrinkers* that do not have to be operated *backward* to effect a *shrink.*

Thus, to *shrink* a board 3 times, we have a choice. We can either put a board through the *3-shrinker forward* or the *3-stretcher backward.* This can develop into

$$9 \times \frac{1}{3} = 9 \div 3$$

Thus, running a board through any kind of machine *backward* is *division;* running it through any kind of machine *forward* is *multiplication.*

After carefully developing the idea with the child that

3 is a *3-stretcher*

and

$\frac{}{3}$ (or $\frac{1}{3}$) is a *3-shrinker*, the operation

$2 \times \frac{}{3}$ (or $2 \times \frac{1}{3}$) has the same meaning as $\frac{2}{3}$.

Then, ideas can follow, such as:

$$3 \div \frac{1}{4} = 3 \div \frac{}{4} = 3 \times 4 = 12$$

or

$$12 \div \frac{2}{3} = 12 \div (2 \times \frac{}{3}) = 12 \times (\frac{}{2} \times 3) = 12 \times \frac{3}{2} = 18$$

FUNCTION MACHINE

OR

WHAT'S MY RULE?

One of the most imortant goals in teaching mathematics is to give the student some problem-solving ability. Surely, a necessary condition to learning to solve problems is to have experience doing just that . . . . . . solving problems. Therefore, even the first grade teacher should give her students beginning experience in problem solving. One nice device for problem solving at any grade level, whether it be the first grade or advanced high school mathematics, is the problem exemplified in *"What's My Rule?"*

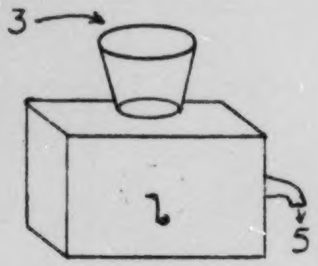
This game requires the participant

- to search for *patterns*
- to seek a solution by *trial and error* (an important part of problem solving)
- to get lots of *vitally important drill* while he is looking for the rule

The *function machine* is an imaginary device which, though not necessary to use in playing *"What's My Rule?"*, nevertheless adds something to the game and simultaneously gives the student a better understanding of the meaning of *function* in mathematics.

A *function machine* is a machine with a *hopper* (3), a *crank* (4), and a *spigot* (5).

A number can be dropped *into the hopper*



the *crank* is turned*
and
another
number
drops out the spigot

It's that simple!

The teacher can design a large variety of *function machines*, each one having its *gears* so arranged as to operate differently on the number received.

The teacher, in describing the machine to the class, simply writes down a set of pairs of numerals, the first of which represents the number dropped into the *hopper*, and the second of which represents the number which comes out the *spigot*.

For example, in an elementary class, the teacher might list several pairs, such as

(1,3) (2,4) (3,5) (6,8) etc. or

In	Out
1	3
2	4
3	5
6	8

* The machine can be modernized by replacing the crank with a push-button.

The first pair of numbers (1,3) are interpreted to mean that when we drop a 1 into the machine, a 3 comes out. Another commonly accepted notation to describe the *Input* and *Output* of the machine is

$f(1) = 3$

or the *function* of 1 is *three*

read "f" of "1" is 3

$f(1) = 3$; $f(2) = 4$; $f(3) = 5$; $f(6) = 8$, etc.

The role of the f in $f(2)$ is merely to identify the *machine* being used. Thus, "g" could be used as the identifying name of another machine that followed a different rule.

As the teacher writes down the pairs of numerals, the child is expected to discover her secret. That is,

HOW DOES THE MACHINE WORK?

A caution here: Encourage the students to avoid giving away the secret (rule); rather encourage the students to give back the second number. (i.e., they give back the second number formed by the rule).

After it is apparent that most students know the rule, the teacher can give the students the second number of the pair and ask them to determine what must have been the number dropped into the machine.

Allowing the rule to be verbalized too early blocks learning, especially by the slower student.

Of course, the possibilities are endless. As the students mature mathematically, the rules can become less and less immediately obvious. Another example of a rule we might ask the students to discover follows. We will call the machine involved -- the "g" machine to distinguish it from the "f" machine described above.

$$(1,3) \quad (2,6) \quad (3,11) \quad (4,18) \quad (5,27) \quad (6,\underline{\quad}) \quad (\underline{\quad},102)$$

The other notation for the same information is given by

$$g(1) = 3,\ g(2) = 6,\ g(3) = 11,\ g(4) = 18,\ g(5) = 27,\ g(6) = \underline{\quad},\ g(\underline{\quad}) = 102.$$

Still another machine might be called the "h" machine and its workings could be described as

$$h(1) = 0,\ h(2) = 2,\ h(3) = 6,\ h(4) = 12,\ h(5) = 20, \text{ etc.}$$

From this point, it is easy to build in the student's mind the concept of a function. For example, if he grasps the rule involved in the "f" and "g" *functions (machines)* just mentioned, he should have no difficulty in interpreting the meaning of $f(g(3))$.

This simply means that a 3 is dropped in the "g" machine, and the number which drops out the spigot drops immediately into the waiting hopper of the "f" machine. Thus

$$f(g(3)) = f(11) = 13$$

That is to say, if a 3 is dropped into the "g" machine, an 11 drops out. This 11 then drops into the "f" machine and a 13 comes out. So,

$$f(g(3)) = 13$$

Let the student next decide if

$$f(g(3)) = g(f(3)).$$

There is not enough information to make that judgment for this particular function set-up. Generally AT LEAST 3 substitutions have to be made in order to understand the functions.

More capable students at the higher levels can be led to exploring such questions as trying to determine for what types of machines, if any,

Is it true that $f(g(x)) = g(f(x))$?

Are there any types of machine for which

$$f(xy) = f(x) \cdot f(y)?$$

Even for the student not ready for such difficult questions, certainly the game of "*What's My Rule?*"* is good drill and good experience in learning to look for patterns, an essential requirement in successful problem solving.

See the next section for more games on "What's My Rule?"

* It is important for the teacher to stress the "*My*" in this because many rules or f-machines can give back the same values up to a point.

GAMES TO USE WITH SECTION ON SETS

WHAT'S MY RULE?

The student observes the numbers that are fed into the machine and the number that comes out of the machine. He tries to determine what the machine does to the number or numbers.

Have the student make up rules for the machine and let the remainder of the students guess the rule.

In

Out

Record of Machine

19	17	10	7	29	37		In
12	10	3				4	Out

2	7	4	13		25	0
4	14		26	20		

3	6	9		2	12	
15	30		5	10		

2	7	1	5	0	8	
4	49	1				

4	7	0	3	5	1
20	56	0	12		

15	3	8	23	1	7
15	27	22			

1	2	3	4	9	
1	3	5			29

Another format for "What's My Rule?"

In	Out
28	14
118	59
2	1
___	73
___	7
___	0

In	Out
6	42
9	90
20	___
4	20
___	12
8	___

In	Out
8	22
5	16
3	___
10	___
1	8
___	24
___	38

In	Out
15	225
0	0
7	49
1	___
10	___
30	___

In	Out
9	8
36	2
1	72
3	___
___	12
___	18
2	___

In	Out
17	4
32	7
47	___
2	1
___	11
12	___

NO TALK GAME: Rules -- If anyone talks, including the teacher, the game is over.
Students come to the board to fill in the blanks. The excitement can be increased by facial expressions of the teacher.
This can be used with any operation.

Give several examples:

3, 4 → 12
6, 6 → 36
4, 8 → 32
6, 5 → ___
7, 8 → ___

In	Out
3, 5	2
7, 1	6
13, 23	10
8, 10	___
4, 10	6
___, 97	13

In	Out
10, 3	8
6, 11	14
4, 1	___
16, 0	8
24, ___	24
___, 14	29
___, ___	1

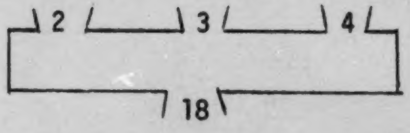
In	Out
5, 9	24
7, 8	29
11, 0	33
1, 0	3
0, 1	1
___, 10	61

WHAT'S MY RULE?

In	Out
5	4
1	0
7	6
9	___
15	___
20	___
n	___

In	Out
9	42
8	30
7	20
6	12
5	___
4	___

In	Out
11	118
15	162
8	___
6	___
3	30
4	___
5	___



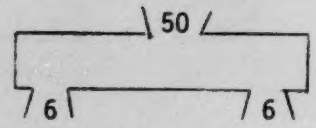
In	Out
2, 3, 4	18
5, 7, 2	28
9, 8, 1	36
2, 2, 2	___
3, 3, 3	___
a, b, c	___

In	Out
2	5
4	11
1	2
5	14
7	___
10	___
n	___

In	Out
2, 3	5
7, 1	8
8, 5	___
3, 6	___
n, x	___

In	Out
2	7
3	10
5	16
7	___
8	___
n	___

In	Out
3, 6	15
1, 5	11
11, 10	31
5, 5	___
___, 7	19
2, 8	___



In	Out
50	6, 6
25	3 1/2, 3 1/2
40	5, 5
60	7, 7
100	___, ___
200	___, ___

○	8	10	3	
2	20	24		
6		64	22	
				49
8				44

As each number comes out, feed back.

A. 16	B. 2
24	4
36	32
54	512
81	___
___	

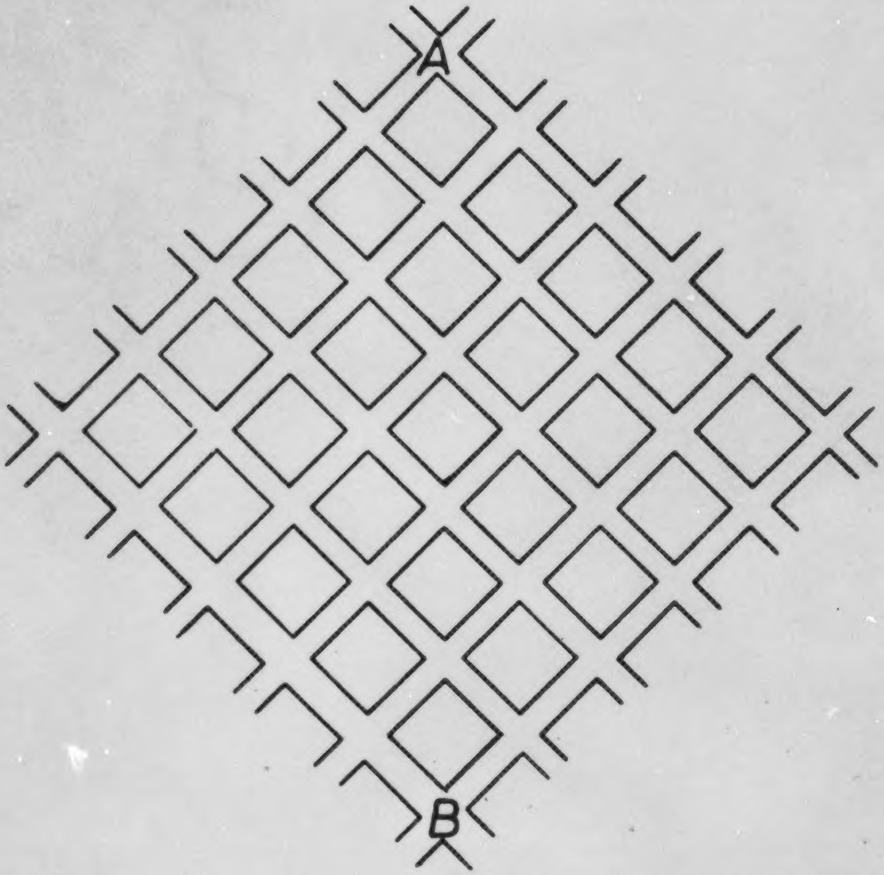
SUGGESTED LABORATORY MATERIALS FOR USE WITH SETS

Marbles
Beads
Pencils
Crayons
Popsicle Sticks
Buttons
Blocks
Paper Plates
Rocks and Shells
String and Yarn
Bottle Caps
Books and Magazines
Wire Models of Geometric Shapes
Pegboard and Wooden Pegs

Cuisenaire Rods
Balloons
Miniature Toys
Empty Cans and Boxes
Embroidery Hoops
Beans
Cardboard or Plastic Shapes
Coins
Flannel Board Materials
Geoboard
Clothespins
Pipe Cleaners
Paper Cups
Paper Sacks

MULTIPLICATION TABLE

X	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
1	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
2	2	4	6	8	10	12	14	16	18	20	22	24	26	28	30	32	34	36	38	40	42	44	46	48	50
3	3	6	9	12	15	18	21	24	27	30	33	36	39	42	45	48	51	54	57	60	63	66	69	72	75
4	4	8	12	16	20	24	28	32	36	40	44	48	52	56	60	64	68	72	76	80	84	88	92	96	100
5	5	10	15	20	25	30	35	40	45	50	55	60	65	70	75	80	85	90	95	100	105	110	115	120	125
6	6	12	18	24	30	36	42	48	54	60	66	72	78	84	90	96	102	108	114	120	126	132	138	144	150
7	7	14	21	28	35	42	49	56	63	70	77	84	91	98	105	112	119	126	133	140	147	154	161	168	175
8	8	16	24	32	40	48	56	64	72	80	88	96	104	112	120	128	136	144	152	160	168	176	184	192	200
9	9	18	27	36	45	54	63	72	81	90	99	108	117	126	135	144	153	162	171	180	189	198	207	216	225
10	10	20	30	40	50	60	70	80	90	100	110	120	130	140	150	160	170	180	190	200	210	220	230	240	250
11	11	22	33	44	55	66	77	88	99	110	121	132	143	154	165	176	187	198	209	220	231	242	253	264	275
12	12	24	36	48	60	72	84	96	108	120	132	144	156	168	180	192	204	216	228	240	256	264	276	288	300
13	13	26	39	52	65	78	91	104	117	130	143	156	169	182	195	208	221	234	247	260	273	286	299	312	325
14	14	28	42	56	70	84	98	112	126	140	154	168	182	196	210	224	238	252	266	280	294	308	322	336	350
15	15	30	45	60	75	90	105	120	135	150	165	180	195	210	225	240	255	270	285	300	315	330	345	360	375
16	16	32	48	64	80	96	112	128	144	160	176	192	208	224	240	256	272	288	304	320	336	352	368	384	400
17	17	34	51	68	85	102	119	136	153	170	187	204	221	238	255	272	289	306	323	340	357	374	391	408	425
18	18	36	54	72	90	108	126	144	162	180	198	216	234	252	270	288	306	324	342	360	378	396	414	432	450
19	19	38	57	76	95	114	133	152	171	190	209	228	247	266	285	304	323	342	361	380	399	418	437	456	475
20	20	40	60	80	100	120	140	160	180	200	220	240	260	280	300	320	340	360	380	400	420	440	460	480	500
21	21	42	63	84	105	126	147	168	189	210	231	252	273	294	315	336	357	378	399	420	441	462	483	504	525
22	22	44	66	88	110	132	154	176	198	220	242	264	286	308	330	352	374	396	418	440	462	484	506	528	550
23	23	46	69	92	115	138	161	184	207	230	253	276	299	322	345	368	391	414	437	460	483	506	529	552	575
24	24	48	72	96	120	144	168	192	216	240	264	288	312	336	360	384	408	432	456	480	504	528	552	576	600
25	25	50	75	100	125	150	175	200	225	250	275	300	325	350	375	400	425	450	475	500	525	550	575	600	625



PROBLEM

How many paths are there from A to B that are equal in distance traveled, and which is the shortest possible path?

NUMBER PATTERNS

Fibonacci
Sequence 1, 1, 2, 3, 5, 8, 13, 21, . . .

Sum of Consecutive Odd Numbers
1 + 3 = 4
1 + 3 + 5 = 9
1 + 3 + 5 + 7 = 16
1 + 3 + 5 + 7 + 9 = 25

Pascal's Triangle

```
            1
          1   1
        1   2   1
      1   3   3   1
    1   4   6   4   1
  1   5  10  10   5   1
_   _   _   _   _   _   _
```

Chain Reaction 1, 4, 7, 10, ___, ___, ___, . . .

What is My Pattern? A) 4, 5, 7, 11, 19, ___, ___, ___, . . .

B) 24, 16, 18, 22, 12, ___, ___, ___, . . .

Sum of Consecutive Even Integers
2 + 4 = 6
2 + 4 + 6 = 12
2 + 4 + 6 + 8 = 20
2 + 4 + 6 + 8 + 10 = 30
2 + 4 + 6 + 8 + 10 + 12 = 42

Sum of Consecutive Integers
1 + 2 = 3
1 + 2 + 3 = 6
1 + 2 + 3 + 4 = 10
1 + 2 + 3 + 4 + 5 = 15
1 + 2 + 3 + 4 + 5 + 6 = 21

DIVISIBILITY RULES

A Number is Divisible by	If
2	the units digit is even.
3	the sum of its digits is divisible by 3. Thus 4605 is divisible by 3 because 15, the sum of its digits, is divisible by 3.
4	its last two figures are divisible by 4. Thus 836 is divisible by 4 becuase 36 is divisible by 4.
5	its units digit is 0 or 5.
6	it is an even number and the sum of its digits is divisible by 3.
7	the ones place is isolated from the number, and multiplied by two, then this number is subtracted from the remainder of the number, beginning with the tens place. If the resulting number is divisible by 7, the original number is divisible by 7, and conversely. The process can be repeated, as shown in Example I.

I. Is 1,667 dividsible by 7?

$$\begin{array}{r} 1\ 6\ 6\ 7 \\ -\ \ \ 1\ 4 \\ \hline 1\ 5\ 2 \\ -\ \ \ 4 \\ \hline 1\ 1 \end{array}$$

Since 11 is not divisible by 7, 1,667 is not divisible by 7.

II. Is 266 divisible by 7?

$$\begin{array}{r} 2\ 6\ 6 \\ -\ 1\ 2 \\ \hline 1\ 4 \end{array}$$

Since 14 is divisible by 7, 266 is also divisible by 7.

DIVISIBILITY RULES

A Number is Divisible by	If
8	the last three figures are divisible by 8. Thus 8144 is divisible by 8 because 144 is divisible by 8.
9	the sum of its digits is divisible by 9.
10	the units digit is 0.
11	the units digit, minus the tens digit, plus the hundreds digit, minus the thousands digit, plus ___, etc., is divisible by 11. Or, if the difference between the sum of the digits in the even places and the sum of the digits in the odd places is 0 or divisible by 11, then the original number is divisible by 11. Thus, 53031 may be recognized as a number divisible by 11 because 1 plus 0 plus 5 equals 6, and 3 plus 3 equals 6, and 6 minus 6 equals 0. The number 162679 is divisible by 11 because 9 plus 6 plus 6 equals 21; 7 plus 2 plus 1 equals 10; and 21 minus 10 equals 11.

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A RATIONALE IN WORKING WITH SIGNED NUMBERS

In the teaching of signed numbers to children in Grades 4-11, the "Postman Stories" model, as devised by the Madison project, has been used.

Postman Stories for Addition and Subtraction

Let us pretend that we live in a city inhabited by people who do things very strangely. For instance, this city has a postman who drops off letters at the different houses quite arbitrarily. He quite often leaves mail at the wrong address and then comes back on a later date to take them back. But what is even more strange, the people of this city really don't seem to mind. They take the letters, read them and then set them aside without noticing to whom they were really meant to be delivered.

Besides receiving letters from friends and relatives, one also receives two other types of mail - checks and bills. The company which employs your father may send his earnings through the mail in the form of a check. Your mother, in her shopping, might not pay cash for the articles she bought. In that case the store will send her a bill for the amount she owes.

Let's put the last two types of letters in a mathematical notation. Then we can use symbols instead of words to describe what has happened. Let us write a check for, say \$6.00, like this: $(^{+}6)$. We will call this "positive 6." A bill for the same amount will be written as $(^{-}6)$ and called "negative 6."

Let us further write the following:

"Deliver a check for \$6.00" as $+\ ^{+}6$

and say "plus a positive 6."

"Deliver a bill for \$6.00" as $+\ ^{-}6$.
"Take back a check for \$6.00" as $-\ ^{+}6$.
"Take back a bill for \$6.00" as $-\ ^{-}6$.

How would you feel, happy or sad, if the postman delivered you a check? (Happy.) How would you feel, happy or sad, if the postman delivered you a bill? (Sad.) How would you feel, happy or sad, if the postman took back a check? (Sad, because now you cannot cash the check.) How would you feel, happy or sad, if the postman took back a bill? (Happy, because now you do not have to pay the bill.)

Now, the housewife has a given amount of money in the bank for her weekly needs, and, even though she doesn't cash any of the checks or pay any of the bills she receives, she does keep track of her financial position by adding the amounts of the checks to and subtracting the amounts of the bills from her balance.

Let us assume that our housewife has \$20 as her balance early Monday morning.

Monday: \$20 --- $^{+}2 + {}^{+}6$

What did the postman do? (He delivered a check for \$2 and a check for \$6.)
Is she happy or sad? (Happy.)
Is our housewife richer or poorer after this transaction? (Richer.)
How much richer? (\$8 richer.)
Let me write "\$8 richer" as $^{+}8$, therefore, $^{+}2 + {}^{+}6 = {}^{+}8$
Then on Tuesday, how much money will she have to work with? (\$28.)

Tuesday: \$28

The postman came and what did he do? $^{-}3 + {}^{-}4$
(He delivered a bill for \$3 and a bill for \$4.)
Happy? Sad? (Sad.)
Is our housewife richer or poorer after this transaction? (Poorer.)
How much poorer? (\$7 poorer.)
Let me write "\$7 poorer" as $^{-}7$, therefore, $^{-}3 + {}^{-}4 = {}^{-}7$
Then on Wednesday, how much money will she have to work with? (\$21.)

Wednesday: \$21

The postman came and what did he do? $^{+}4 + {}^{-}6$
(He delivered a check for \$4 and a bill for \$6.)
Happy? Sad? (Sad.)
Richer? Poorer? (Poorer.)
How much poorer? (\$2 poorer.)
How should I write that she is \$2 poorer? ($^{-}2$) $^{+}4 + {}^{-}6 = {}^{-}2$
Then on Thursday, how much money will she have to work with? (\$19.)

Thursday: $19

The postman came and what did he do? $^{+}7 + {}^{-}3$
(He delivered a check for $7 and a bill for $3.)
Happy? Sad? (Happy.)
Is our housewife richer or poorer after this transaction? (Richer.)
How much richer? ($4 richer.)
How should I write "$4 richer"? ($^{+}4$) $^{+}7 + {}^{-}3 = {}^{+}4$
Then on Friday, how much money will she have to work with? ($23.)

Friday: $23

The postman came and what did he do? $^{+}3 - {}^{+}6$
(He delivered a check for $3 and took back a check for $6.)
Happy? Sad? (Sad.)
Which part made her happy? (Delivering the check for $3.)
Which part made her sad? (Taking back the check for $6.)
Is our housewife richer or poorer after this transaction? (Poorer.)
How much poorer? ($3 poorer.)
How should I write "$3 poorer"? ($^{-}3$) $^{+}3 - {}^{+}6 = {}^{-}3$
Then on Saturday, how much money will she have to work with? ($20.)
Her balance is the same now as it was when she started. Why? (All bills and checks cancel; discuss.)

Saturday: $20

The postman came and what did he do? $^{-}4 - {}^{-}6$
(He delivered a bill for $4 and took back a bill for $6.)
Happy? Sad? (Happy.)
Which part made her happy? (Taking back the bill for $6.)
Which part made her sad? (Delivering the bill for $4.)
Is our housewife richer or poorer after this transaction? (Richer.)
How much richer? ($2 richer.)
How should I write "$2 richer"? ($^{+}2$) $^{-}4 - {}^{-}6 = {}^{+}2$
Then on the next delivery day, Monday, how much money will she have to work with? ($22.)

Monday: $22

The postman came and what did he do? $^{+}7 - {}^{-}4$
(He delivered a check for $7 and took back a bill for $4.)
Happy? Sad? (Happy.)
Which part made her happy? (Both transactions.)
Which part made her sad? (Neither.)
Is our housewife richer or poorer after this transaction? (Richer.)
How much richer? ($11 richer.)
How should I write "$11 richer"? ($^{+}11$) $^{+}7 - {}^{-}4 = {}^{+}11$
Then on Tuesday, how much money will she have to work with? ($33.)

Tuesday: $33

The postman came and what did he do? $^{-}8 + {}^{+}5$
(He delivered a bill for $8 and took back a check for $5.)
Happy? Sad? (Sad.)
Which part made her happy? (Neither.)
Which part made her sad? (Both transactions.)
Is our housewife richer or poorer after this transaction? (Poorer.)
How much poorer? ($13.)
How should I write "$13 poorer"? ($^{-}13$) $^{-}8 + {}^{+}5 = {}^{-}13$
Then on Wednesday, how much money will she have to work with? ($20.)
Her balance is the same as it was when she started. Why? (Again, checks and bills cancel; discuss.)

Wednesday: $20

The postman came and what did he do? $^{+}3 - {}^{-}3$
(He delivered a check for $3 and took back a bill for $3.)
Happy? Sad? (Happy.)
Is our housewife richer or poorer after this transaction? (Richer.)
How much richer? ($6 richer.)
How should I write "$6 richer"? ($^{+}6$) $^{+}3 - {}^{-}3 = {}^{+}6$
Then on Thursday, how much money will she have to work with? ($26.)

Thursday: $26

The postman came on this day, like always, but this time when I asked the housewife what happened, this is what she told me:

$$^{+}6 + \square = {}^{-}8$$

What had to happen here so that she would end up $8 poorer? (He delivered a check for $6 and a bill for $14.) Then on Friday, how much money will she have to work with? ($18.)

Friday: $18

This is what she told me happened on Friday:

$$^{+}3 - \square = {}^{+}5$$

What happened to make her $5 richer? (He delivered a check for $3 and took back a bill for $2.) Then on Saturday, how much money will she have to work with? ($23.)

Saturday: $23

What would have had to happen here? $\square - {}^{-}4 = {}^{-}10$

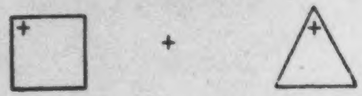
(He delivered a bill for $14 and took back a bill for $4.)
How much money will be available next Monday? ($13.)

Monday: $13

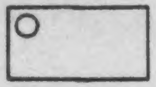
What would have had to happen here? $\square - {}^{-}4 = {}^{+}10$

(He delivered a check for $6 and took back a bill for $4.)

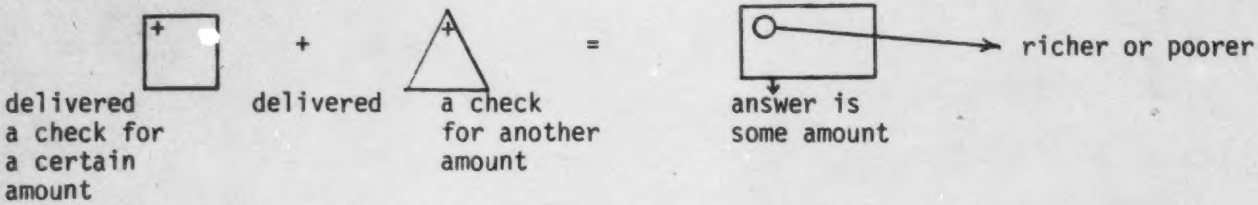
What if the postman came, delivered a check for a certain amount, and delivered another check for another amount? Let me symbolize the last sentence in this manner.



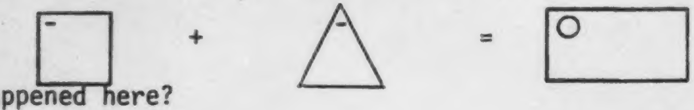
We know the answer will be a certain amount -- but I'm not really interested in exactly how much it is. What I am interested in is whether our housewife will be richer or poorer. I will use a rectangle as the answer to our problem -- and



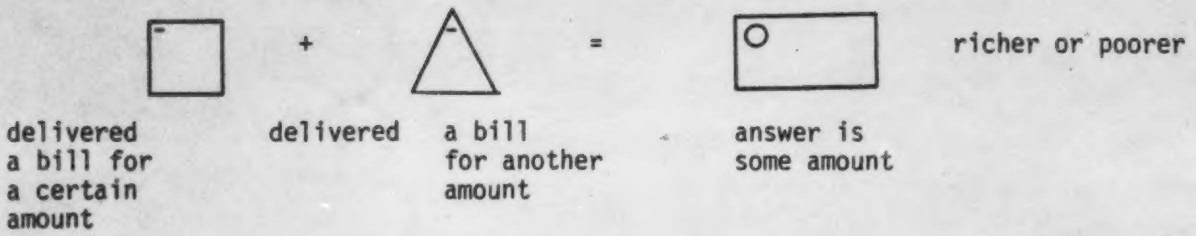
your task will be to tell me what symbol to place in the circle in the rectangle to show whether she is richer or poorer.


+
+
+
=
richer or poorer
delivered a check for a certain amount
delivered
a check for another amount
answer is some amount

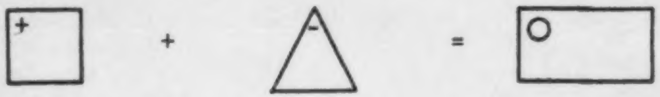
Is she richer or poorer? (Richer.)
What symbol should I put in the circle? (+)



What happened here?

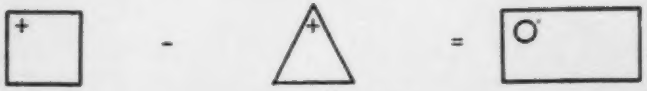

-
+
-
=
O
richer or poorer
delivered a bill for a certain amount
delivered
a bill for another amount
answer is some amount

Is she richer or poorer? (Poorer.)
What symbol should I place in the circle? (-)



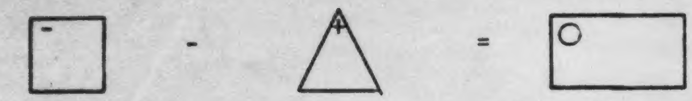
What happened here? (A check for a certain amount and a bill for another amount were delivered.)
Is our housewife richer or poorer? (You cannot tell.)
Why? What would have to happen to make her richer? To make her poorer?

(Richer - the amount of the check "$\boxed{+}$" would have to be larger than the amount of the bill "$\overset{-}{\triangle}$".
Poorer - the amount of the bill "$\overset{-}{\triangle}$" would have to be larger than the amount of the check "$\boxed{+}$".)

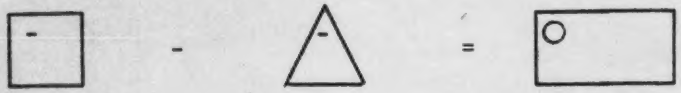


What happened here? (A check for a certain amount was delivered and a check for an amount was taken back.)
Is our housewife richer or poorer? (You cannot tell.)
Why? What would have to happen to make her richer? Poorer?

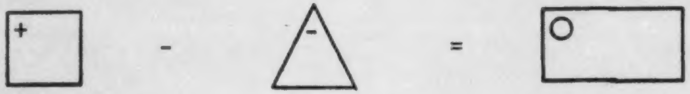
(If the "$\boxed{+}$" check is more than the "$\overset{+}{\triangle}$" check, she will be richer. If the "$\boxed{+}$" check is less than the "$\overset{+}{\triangle}$" check, she will be poorer. If the "$\boxed{+}$" check and the "$\overset{+}{\triangle}$" check are the same amounts, she will be neither richer nor poorer. She will have the same as she started out with that morning.)



Can you tell me what happened here? (A bill for a certain amount was delivered and a check for an amount was taken back.)
Can you tell me if our housewife is richer or poorer from this transaction? (Yes, she is poorer.)
What symbol shall I place in the circle? (-)



What symbol shall I place in this circle? (You cannot tell.)
If we want to make our own housewife richer, what would have to happen?
(The "△" bill would have to be more than the "$\boxed{-}$" bill.)
If she ends up poorer, what would have to happen?
(The "$\boxed{-}$" bill would have to be more than the "△" bill.)
Is it possible for our housewife to break even -- be neither richer nor poorer?
(Yes, the "$\boxed{-}$" bill and the "△" bill would have to be for the same amounts.)



What symbol shall I place in this circle? (+)
Does this mean that she will be richer or poorer? (Richer.)

The story, though odd in plot, gives the children a rationale in working with signed numbers. The solution of $^-3 + ^-6 = ^+9$ will just not hold water because the children can really feel that if two bills are delivered, a person must end up poorer -- not richer.

A RATIONALE IN WORKING WITH SIGNED NUMBERS - REVISITED

The previous article covered the addition and subtraction of signed numbers. A model was constructed using a postman who quite often delivered mail to the wrong addresses. He delivered checks and bills, and since he made errors in these deliveries from time to time, he would come back to the houses where the mistakes had been made and take back the erroneously delivered checks and bills.

This article will cover the multiplication and division of signed numbers.

The following model was used to show the delivery and taking back of these checks and bills.

Check for \$6
$^{+}6$ read "positive six"

Bill for \$6
$^{-}6$ read "negative six"

Delivered a check for \$6
+ $^{+}6$ read "plus a positive six"

Delivered a bill for \$6
+ $^{-}6$ read "plus negative six"

Took back a check for \$6
- $^{+}6$ read "minus a positive six"

Took back a bill for \$6
- $^{-}6$ read "minus a negative six"

Thus, $^{+}4$ + $^{-}5$
delivered a check for \$4
delivered a bill for \$5

This would make the housewife sad, for she would be \$1 poorer at the end of this transaction ($^{+}4$ + $^{-}5$ = $^{-}1$).

The model for the postman stories has to be changed for the multiplication operation. This is regretable, but definitely necessary.

In the procedure for finding the area of, let us say, a rectangle, one multiplies the length by the width (Figure 1).

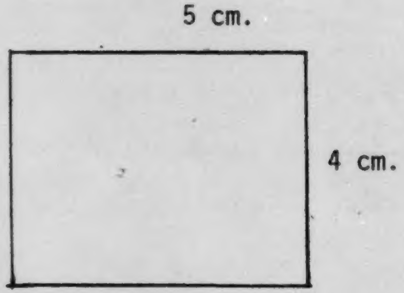


Figure 1

For this example, the area would be written as:

$A = l \times w$

$A = 5 \text{ cm.} \times 4 \text{ cm.}$

This can be carried out to:

$A = 5 \times 4 \times \text{cm.} \times \text{cm.}$

$A = 20(\text{cm.})^2$

A = 20 square centimeters

which literally means that one can find 20 "one-centimeter squares" in this rectangle (Figure 2).

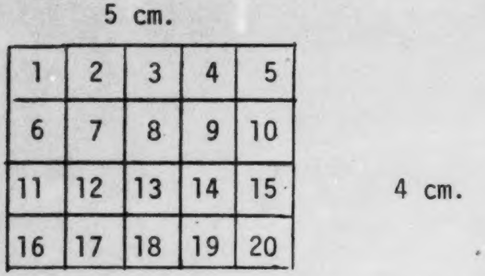


Figure 2

The sentence $^{+}5 \times {}^{+}4$ (with our past meaning) would state: "a check for $5 times a check for $4," which might be extended to

5 x 4 x dollars x dollars

20 (dollars)^2

20 square dollars

which has no meaning to anyone. This is analogous to stating, "5 boys x 4 boys is 20 square boys." Again, no meaning. Since the meaning is not clear, a change in the model is necessary.

Let us assume the postman came and delivered the following:

$^{+}4 + {}^{+}4 + {}^{+}4 + {}^{+}4 + {}^{+}4.$

Will this transaction make our housewife richer or poorer? (Richer.)
How much richer? ($20.)
What symbol may I use to show she is richer? (+)
How will the answer to this problem look? ($^{+}20$.)

Is there another way to say what the postman delivered other than "He delivered a check for $4, and he delivered a check for $4, and another check for $4, and another check for $4, and another check for $4? (He delivered 5 checks for $4 each.)

An addition operation was used for:

$^{+}4 + {}^{+}4 + {}^{+}4 + {}^{+}4 + {}^{+}4.$

What operation is being used when you state, "He delivered 5 checks for $4 each"? (Multiplication.) This can be written with symbols as $5 \times {}^{+}4$ where the first number (5) tells us how many, and the second number ($^{+}4$) tells us the amount and whether it was a check or a bill.

How would you write the following as a multiplication problem?

$^{-}4 + {}^{-}4 + {}^{-}4 + {}^{-}4 + {}^{-}4$

$(5 \times {}^{-}4)$

This seems to follow from before.

What if instead of delivering five checks for $4 each, the postman had taken back five checks for $4 each? (A symbol would have to be made to show whether mail was delivered or taken back.)

Since the first number tells us how many, let's place a positive sign (+) with a number to show a delivery and a negative sign (-) to show a taking back. Then our first problem would be written as:

$^{+}5 \quad x \quad {}^{+}4$

Delivered five checks $4 each

and our second as:

$^{+}5 \quad x \quad {}^{-}4$

Delivered five bills $4 each

What would our third problem look like?

$(^{-}5 \quad x \quad {}^{+}4)$

What happened here?
$^{-}5 \quad x \quad {}^{-}4$

(Postman took back five bills for $4 each.)

What happened here?
$^{+}3 \quad x \quad {}^{+}6$

(Postman delivered three checks for $6 each.)

Is our housewife poorer or richer? (Richer.)

How will the answer be written? $(^{+}18)$
$(^{+}3 \quad x \quad {}^{+}6 \quad = \quad {}^{+}18)$

What occurred here?
$^{+}3 \quad x \quad {}^{-}6$

(Postman delivered three bills for $6 each.)

Is our housewife richer or poorer? (Poorer.)

How will the answer be written? $(^{-}18)$
$(^{+}3 \quad x \quad {}^{-}6 \quad = \quad {}^{-}18)$

And here?
$^{-}3 \quad x \quad {}^{+}6$

(Postman took back three checks for $6 each.)

Is she richer or poorer? (Poorer.)

How will we write the answer? $(^{-}18)$
$(^{-}3 \quad x \quad {}^{+}6 \quad = \quad {}^{-}18)$

And here? $^{-}3 \quad x \quad {}^{-}6$

(Postman took back three bills for $6 each.)

Is she richer or poorer? (Richer.)

How will we write our answer? $(^{+}18)$
$(^{-}3 \quad x \quad {}^{-}6 \quad = \quad {}^{+}18)$

$\square \times {}^{-}4 = {}^{-}12$

In the sentence above what is it that we know?

(Bills are for \$4 each and she ends up \$12 poorer.)

What is not given?

(The number of bills and whether they were delivered or taken back.)

What do you think occurred?

(3 items were delivered.)

What will we have to put in the box to show this?

$({}^{+}3)$

What will our complete sentence then look like?

$({}^{+}3 \times {}^{-}4 = {}^{-}12)$

What will you place in the box to make this sentence true?

$\square \times {}^{+}5 = {}^{-}30$

$({}^{-}6)$

What does that mean? (6 items were taken back.)

${}^{+}4 \times \square = {}^{+}28$

In the sentence above what it is that we know?

(4 items were delivered and she is \$28 richer.)

What is not given?

(The amount and whether it was a check or a bill.)

What do you think belongs in the $\square$?

$({}^{+}7)$

What does this mean?

(They were checks and were for \$7 each.)

What does our complete sentence look like?

$({}^{+}4 \times {}^{+}7 = {}^{+}28)$

What do you think belongs in the box for this sentence?

${}^{-}6 \times \square = {}^{+}24$

$({}^{-}4)$

What does that mean? (Bills for \$4 each.)

Were we given information as to whether these bills were delivered or taken back?

(yes)

Where? (The negative sign in front of the 6 means they were taken back.)

What goes in the box for the following?

(1) $^{-}5 \times \square = {}^{+}10$ ($^{-}2$)

(2) $^{+}3 \times {}^{-}8 = \square$ ($^{-}24$)

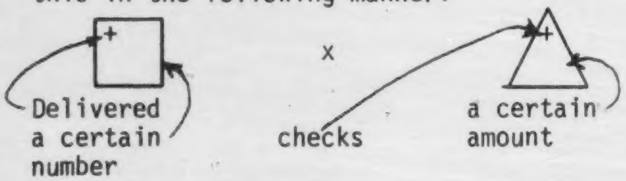
(3) $\square \times {}^{-}2 = {}^{-}18$ ($^{+}9$)

(4) $\square \times {}^{-}3 = {}^{+}9$ ($^{-}3$)

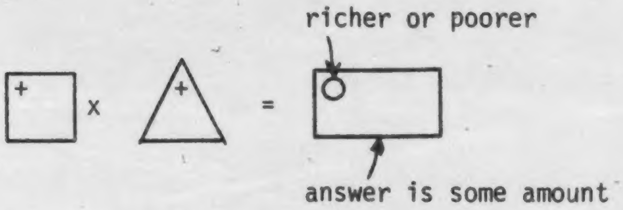
(5) $^{+}8 \times {}^{+}2 = \square$ ($^{+}16$)

(6) $^{+}4 \times \square = {}^{-}16$ ($^{-}4$)

If our postman came and delivered a certain number of checks for a certain amount, we might symbolize this in the following manner:

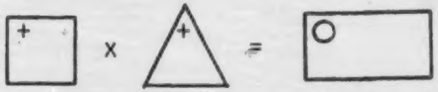


We know the answer will be some amount -- but I'm not really interested in exactly how much it is. What I am interested in is whether our housewife will be richer or poorer I will use a rectangle ▭ as the answer in our problem, and your task will be to tell me what symbol to place in the circle in the rectangle to show whether she is richer or poorer.

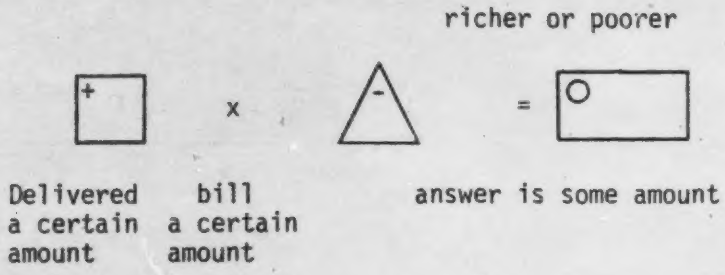


Is she richer or poorer? (Richer.)

What sign should I place in the circle? (+)



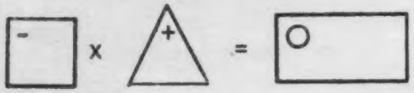
What happened here?


richer or poorer
+
x
-
=
Delivered a certain amount
bill a certain amount
answer is some amount

Is she richer or poorer? (Poorer.)

What symbol should I place in the circle? (-)

How about here, what happened?

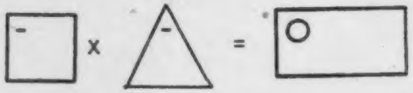


(A certain number of checks for a certain amount were taken back.)

Is she richer or poorer? (Poorer.)

What symbol should I place in the circle? (-)

This one -- what happened?



(A certain number of bills for a certain amount were taken back.)

Richer or poorer? (Richer.)

What symbol? (+)

Through this type of discussion, the "rules" for multiplication of signed numbers come about naturally. Also, a workable explanation for the reason why "a negative times a negative is a positive" is provided by this model. Try it!

MATRICES

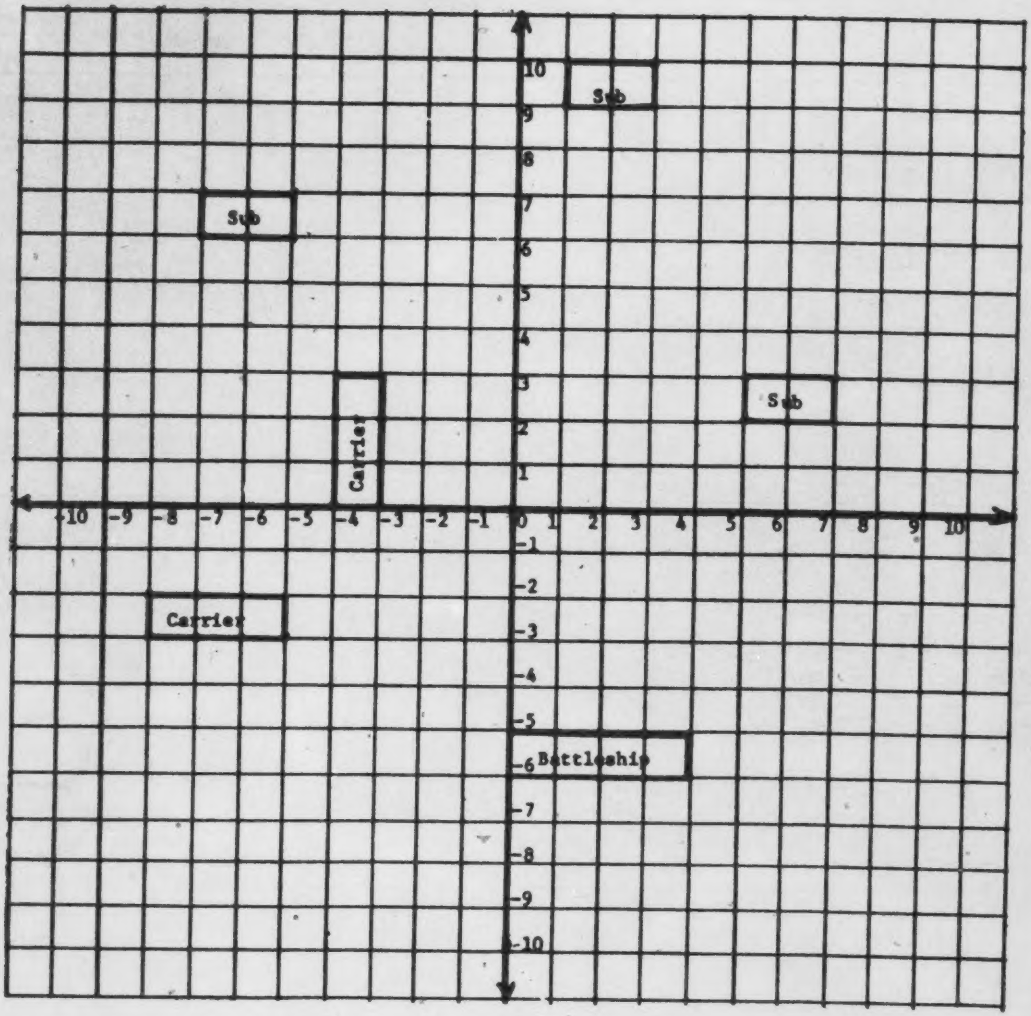
For Operations With Integers

				+3	0	3	6	9	
				+2	0	2	4	6	
				+1	0	1	2	3	
				0	0	0	0	0	
	-3	-2	-1	X	0	+1	+2	+3	
				-1					
				-2					
				-3					

				+3	3	4	5	6	
				+2	2	3	4	5	
				+1	1	2	3	4	
				0	0	1	2	3	
	-3	-2	-1	+	0	+1	+2	+3	
				-1	-1	0	1		
				-2	-2	-1	0		
				-3	-3	-2	-1		

Have students fill the remainder of the matrices by observing the pattern that is forming. Transfer to graph paper and extend.

BATTLESHIP


Sub
Sub
Sub
Carrier
Carrier
Battleship
-10
-9
-8
-7
-6
-5
-4
-3
-2
-1
0
1
2
3
4
5
6
7
8
9
10

633

RULES OF BATTLESHIP GAME:

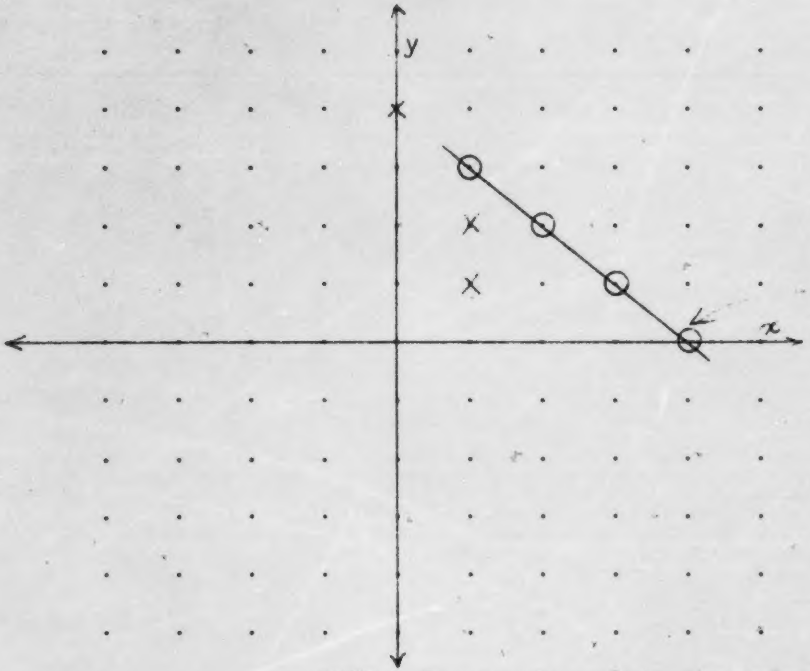
Ships to be hidden throughout the co-ordinate plane; 1 battleship (10 points); 2 carriers (8 points each); and 3 submarines (6 points each.)

The object of the game is to locate the address and capture as many ships as possible. No character can be captured unless all points have been located. When his complete address has been located, that ship has been captured.

The class is divided into two teams. Each team takes turns choosing the pairs on the coordinate plane. A team is allowed just two calls at any one time. The teacher circles each point as it is called. After the point for each ordered pair is circled, the player is informed whether he has been successful in locating one of the points surrounding a ship. The team that captures the most characters wins the game.

VARIATION: Have each team (or individual) locate the ships on a coordinate plane. Then the teams (or individual) that manages to locate all of their opponents' ships first is the winner.

TIC TAC TOE



Divide the class into two teams. A member from Team A gives the first ordered pair of numbers (x,y). The teacher plots the point ⊙.

Then a member from Team B gives an ordered pair. The teacher plots this point with a different symbol (x).

Then the next member from Team A--then the next member from Team B until one team has four points in a straight line (horizontally, vertically, or diagonally).

There is to be no coaching from other team members or the team loses its turn.

NOTE: The teacher plots the point that is first given by the student--the student cannot change his mind.

The upper right-hand quadrant can be used for the lower grades where only positive numbers and zero have been introduced. The game can then be extended to use the other three quadrants whenever the integers have been introduced. This makes an exciting game until the students have learned how to block each other. The rules should then be modified, e.g., line up any three points, etc.

MEASUREMENT AND ESTIMATION

DEVELOPING A LABORATORY

A measurement laboratory can be developed to include all the material needed to provide meaningful experiences in measurement for all the children in a classroom.

THE MATERIAL

Thermometer (Celsius)
Kitchen or baby scales
cm^3 blocks
dm^3
cm^2 paper
Milk containers (liter, 2 liter, 4 liter, etc.)
4 liter jug
Weights (of sand, etc.)
Baby bottle with milliliter graduations
Kitchen measuring cup (metric)
Coffee cup
Eye Dropper
Funnel
5 milliliter spoon
10 milliliter spoon
Ruler
Meterstick
Tape measure (metric)

PROCEDURE FOR USING THE LABORATORY MATERIAL

Problem directions are typed on 8cm x 12cm cards, or printed on 8cm x 12cm cards, depending on the grade level. Each card directs the child to go to the laboratory table and do a specific task. He then reports his findings to the teacher either verbally or on paper.

SAMPLE PROBLEMS:

1. Fill a glass with cold water at the sink. Put the thermometer in the water and leave it there until you think the mercury has gone up or down as far as it is going to go. Record the temperature on your paper. Now fill the glass with hot water and repeat the process.

2. Place the cm^2 tiles on top of a shoe box until it is covered. How many cm^2 does it take to cover the box?

3. Take the dm^3 to the filing cabinet. See if you can figure out how many dm^3 the cabinet would hold.

4. See how much your arithmetic book and reading book weigh together.

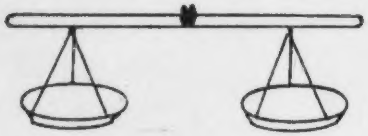
5. Find the milk carton that holds 500 milliliters. Fill it with water and pour the water into the 4 liter jug. How many 500 milliliters does the jug hold?

6. Put cm^3 in the box marked "A" until it is full. How many does it hold? How many layers does it hold?

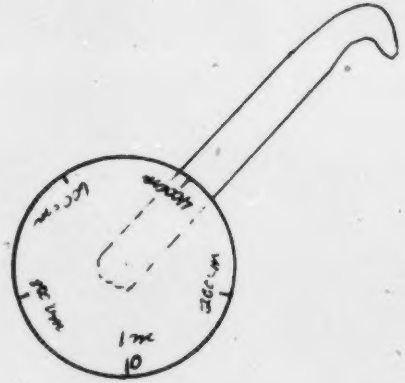
MEASUREMENT AND ESTIMATION

1. Make a square meter from wrapping paper or newsprint. Make 100 square dm of a second color to cover one square meter. Rule one square dm into square cm.

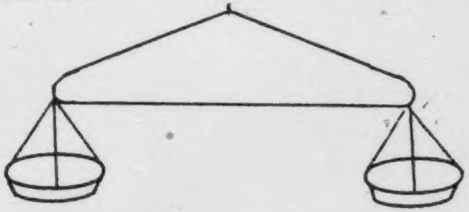
2. Make a simple balance with a stick and pans from paper pie plates suspended by string.



3. A trundle wheel is a measuring device the shape of a wheel which can be rolled along the ground to measure distance. They can be constructed from heavy cardboard or plywood. They may be marked off in terms of a meter.



5. You can make a simple balance by attaching 2 containers, such as the aluminum dishes frozen meat pies are sold in, by wire or string to each end of a wire coat hanger. Use three strings about 15 cm long to attach each container.



6. CARDBOARD THERMOMETER. Make a large cardboard thermometer for demonstration purposes. Mark a Celsius scale on the cardboard and have a moveable ribbon or cord, a part of which is red, to represent the liquid in the thermometer.

GAMES

GUESSING AND CHECKING

Play a game of guessing and checking the weights of different children and of different objects in your room. Let several children lift an object in turn. Each will guess the weight and record his guess. Then the object is weighed to see which children came closest to the correct weight. Children are encouraged to use good judgement in making guesses. For example, if a child who has just been weighed stands beside the child who will be weighed next, there is some basis for comparison.

If children are to improve in the ability to make approximate estimates, they must discover and keep in mind that large objects are not necessarily heavy nor are small ones always light. They must learn to estimate weight by actually lifting objects or by some visual comparison of similar objects.

"Guessing and checking" can also be played with measuring. Mark off a meter along the floor to serve as a model measure. The door, a window, the chalkboard, may be measured and marked as other model measures. Provide rulers, metersticks, and tape for measuring stationary objects. Guess the width,length, and height of various objects, record the guesses, and measure to determine correct measurements.

Many of our decisions with regard to number experiences are reached, not by the pencil and paper method, but by rapid, rough calculations which give a working estimate of the amount. Any number learning which leads the child to improve his ability to reason sensibly and arrive at a fairly accurate estimation will prove of infinite worth to him.

HAULING LUMBER

A miniature lumber yard is set up and the lumber (wooden sticks) is bundled into tens. This enabled the lumber man to ship his lumber more easily. A toy truck was used for making deliveries. The boy, pretending that he was the truck driver, did the bundling, the loading, and the unloading, and kept account of his deliveries. At the beginning he drew heavy black lines to stand for each bundle of ten and thin lines to stand for the single sticks. Payment was made in pennies only, or in dimes for the bundles and pennies for the single sticks. Finally, figures were used to sum up the number of bundles and single sticks which were checked with the number of dimes and pennies.

MEASUREMENT AND ESTIMATION

SHOPPING TRIP

Cut grocery store ads from the newspaper. Divide the class into teams. Make a different shopping list for each team. Include on each list the amount of money the team has to spend.

Post the various ads in different parts of the room and let the teams "go shopping." Team members may discuss their purchases with one another, but keep a record separately so they can check together later to avoid errors.

A shopping list will include the need for the four operations. If an ad reads "2 cans for 26¢" and only one can is on the list, the children must divide by 2. If something costs 32¢ per box and the list reads "4 boxes," the children would multiply. Interesting discussions will arise from "3 for $1.00" ad.

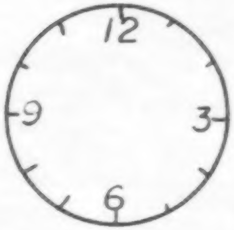
When a team completes its list, each child totals his own list, breaks down the change he will receive into the proper coins, and compares his result with the other team members.

The teacher should check each team report for accuracy. She can then inform each team of its progress in relation to the last "Shopping Trip."

HOW MUCH MONEY HAVE I? A child stands holding in his closed hand toy money. He turns to a pupil and says, "I have a nickel and three pennies. How much money have I?" If the child answers, "You have eight cents," he takes a group of coins, closes his hand, and tests another pupil.

A CALENDAR GAME: One week ago--one week from now game. Students give a date one week ago and one week from now. What day is 4 weeks from your birthday? What day is one month from your birthday?

THE CLOCK GAME: Draw a clockface on the board--labeled as below. Where would the big and little hands be at one o'clock? Have a volunteer draw the hands where they would be an hour later. Keep a record of the results.



one o'clock.......	1:00
an hour later.....	2:00
30 minutes later..	2:30

PICK A LUNCH: From a menu the students select what they would like and find the total cost. This may be expanded to have them tell what coins they would get in change from $1.00.

MENU

Sandwiches	
Cheese	25
Ham	30
Peanut Butter	20
Hamburger	30
Hot Dog	20
Tuna	25
Beef	35

Beverages	
Milk	12
Milk Shake	25
Root Beer	10
Orange	10
Coffee	10
Tea	10

Soups	
Chili	30
Vegetable	25
Bean	20

Desserts	
Ice Cream	15
Cake	20
Pie	20

CHOOSING BLINDLY: Supply the class with a discarded purse containing 10 dimes and 10 pennies. Ask one child to come up and take out some coins without looking. Let him pick someone to identify the chosen coins and to write the value of the set on the board. The second child should then choose another to tell how many coins are left in the purse and to write their value on the board. A fourth child, selected by the third, should tell which set is larger, return the coins to the purse, and choose another child to repeat the process.